NUMS
Abstracts

Nicholas Anderson, San Francisco State University
Eta-Quotients of Prime Or Semiprime Level And Elliptic Curves

This presentation will highlight the results proven during the Oregon State University REU this past summer. Newman's theorem, often referenced through Ken Ono's "The Web of Modularity" as Theorem 1.64, is ubiquitous in the study of eta-quotients as "nice" modular forms. The theorem lays down sufficient conditions for an eta-quotient to be a modular form. In our paper, we prove that in the case where the level of the modular form is coprime to 6, that these conditions are necessary. Moreover, we use this, along with several other corrections, to rectify some recent results published by Arnold-Roksandich, James, and Keaton. We further generalize the results in their paper to semiprimes, and connect these results to computing bases of vector spaces of modular forms composed entirely of linear combinations of eta-quotients. This presentation will focus on our motivating questions, and the techniques we used to prove some of our results.

Alyssa Arnaud, University of Portland
Modeling Bear Populations in North Carolina

The connection between human activities and ecological changes is a complex web of interactions with non-linear dynamics. We use a data driven approach to study if the increase in human population caused significant changes in the environmental systems to cause the female black bears to start birthing earlier with more births. We analyze the trends of animal reproduction cycle with respect to the geographical and human population dynamics of the North Carolinas counties. The geography divides the counties into the coastal, low lying, and mountainous and we use the Bhattacharya variance measure to group the counties by population trends into increasing, decreasing or constant. The available dataset reports the reproductive information of bears harvested in North Carolina between 1969 and 2012 with over 14,000 reported data points. We use polynomial regressions and statistical analysis to build the correlations to measure the mutual dependency and variance between the human population increase and birthing cycle of perturian female bears.

Emma Bidwell, George Fox University

Applications of the Group J, the Voice-Leading Tonnetz, and Diminishing Groups in Schubert's B-flat Major Sonata

Motivated by Schubert's Piano Sonata in B-flat Major, D. 960, we expand knowledge of the J group developed by Fiore and Noll. In the spirit of David Lewin, we use the PLR-group to make both global and local maps of the sonata, following Richard Cohn. We use the Structure Theorem of Fiore-Noll to find J group operations that realize some of these musical motions while preserving voice ordering. As an enrichment of the neo-Riemannian Tonnetz we develop a voice leading Tonnetz for the J group as a simplicial set, rather than simplicial complex. As we explore the topological structure of our Tonnetz, we observe the elements of the extended J group that preserve Cohn's hexatonic set. Finally, we propose three new groups of singular matrices that accomplish major to diminished triad movement, motivated by Schubert's use of diminished triads. New mathematical theorems are: the geometric realization of the voice leading Tonnetz is a 6-fold cover of the neo-Riemannian Tonnetz, there does not exist a matrix that sends diminished chords to major chords compatible with transposition, and the determination of the subgroup of $\Sigma 3 \ltimes J$ appropriate for the voice leading Tonnetz.

Justin Bloom, Oregon State University

The Drawing Trick, and its implications in higher dimensions

In John Meier's text "Groups, Graphs, and Trees", he describes what is called "The Drawing Trick", a method to construct the Cayley graphs of finitely generated groups. First, there needs to be some geometric representation for a group. More precisely, the group must act on a mathematical object, and for the purposes of "The Drawing Trick" there needs to be something to draw, which is why some symmetrical geometric object is preferred. Then by theorems in abstract algebra you can find a clever choice for a point on that object, and trace where it moves after applying elements of the group in question. In my presentation I will give a detailed walk through of how you can use the Drawing trick on a four dimensional object called a regular 5-simplex. Afterwards I'll provide brief explanations about how the resulting Cayley graph is related to combinatorics. I have an accompanying animated visual aid of the Cayley graph that I generated with a python script. My intentions with the visual aid are that some of the inherent difficulties of visualizing a four dimensional object can be remedied.

Maxine Calle, Reed College

Putting the "k" in Curvature: k-Plane Constant Curvature Conditions

Differential geometry studies properties of manifolds using the tools of calculus and linear algebra, and we can characterize the local behavior of manifolds by studying representative model spaces. Studying curvature in this context allows us to generate representative numbers for the model spaces, which in turn characterizes the manifolds. This research generalizes the curvature invariants known as constant sectional curvature and constant vector curvature. While both conditions are well understood considering 2-plane sectional curvature in 3-dimensional model spaces, little is known with regard to higher dimensions. We generalize these curvature conditions using k-plane scalar curvature in Riemannian model spaces of arbitrary finite dimension, and many results are generalizations of known aspects of 2-plane constant curvature conditions. By studying these k-plane curvature invariants, we can further characterize model spaces by generating representative numbers for various subspaces.

Benjamin Clark, University of Washington Bothell
On Polynomials that Preserve all Nonnegative Matrices of a Fixed Order

The nonnegative inverse eigenvalue problem (NIEP) is to characterize the spectra (i.e., list of eigenvalues) of entrywise nonnegative matrices. More specifically, given a finite list $S = \{s_1, ..., s_n\}$ of complex numbers, the NIEP asks for necessary and sufficient conditions such that S is the spectrum of an $n \times n$ entrywise-nonnegative matrix. In further pursuit of a solution to the NIEP, Loewy and London [MR0480563; Linear and Multilinear Algebra, 6(1), 8390, 1978/79] posed the problem of characterizing all polynomials that preserve all nonnegative matrices of a fixed order. If P_n denotes the set of all polynomials that preserve all $n \times n$ nonnegative matrices, then it is clear that all polynomials with nonnegative coefficients belong to P_n. However, it is known that P_n contains polynomials with negative entries. In this work we present partial results for P_n for all orders and on polynomial functions that preserve the nonnegativity of certain structured matrices, including circulant matrices and Jordan blocks. Implications for further research are discussed.

Quinton Cook, Western Washington University
The ordinal and cardinal numbers

An overview of the construction of the ordinal and cardinal numbers on an intuitive level. The talk will build up to two main theorems involving the ordinals, the first of which is the well ordering theorem, which says that any set can be well ordered. The second theorem is that any well-ordered set is order isomorphic to a unique ordinal number. Time permitting a short history of the concept of ordinal and cardinal numbers will be given.

Kimberly Cote, University of Washington Tacoma
Optimization Art

Artwork, such as photomosaics and domino portraits, can be created by modelling the assignment of tiles as an instance of the General Assignment Problem. The algorithm of the Hungarian Method was studied and applied to the problem of optimizing the assignment of tiles in an $n \times n$ photomosaic. It is shown that the method can also be applied to the domino portrait's more intricate objective function.

Reyna Garduno, University of Washington Bothell
Enhancing the Spectral Subgradient Method

The Spectral Subgradient method was proposed to minimize convex functions that were not differentiable at minimizers. It combines the classical subgradient approach and a nonmonotone linesearch with the spectral step length, which does not require any previous knowledge of the optimal value. Although this algorithm does a respectable job in obtaining a solution, it performs a high number of function evaluations, leading to heavy consumption of CPU time. The purpose of this work is to enhance and further develop this algorithm, by incorporating new techniques that include Tan's smoothing technique and Camerini's heavy-ball approach. These techniques are meant to improve the numerical performance of the spectral step and the direction by lowering the number of function evaluations while keeping the same quality of the solution. Numerical results showed improvements on the number of function evaluations, while losing a portion of the quality of the solution.

Brad Halter, University of Washington Bothell

Applying Spectral Projected Gradient and Simulated Annealing Optimization for CT Image Reconstruction

We apply the hybrid optimization method of Guerrero (et al.) to CT image reconstruction. CT image reconstruction can be posed as the problem of minimizing $|Az - b| + \lambda T(z)$, where $T(z)$ is a Tikhonov regularization term to account for ill-conditioning of the Hessian. The hybrid method uses the spectral projected gradient method to compute the optimal regularized solution, along with simulated annealing to simultaneously determine the regularization parameter λ. The infinity norm of the spectral projected direction is used by simulated annealing as the energy function to determine a suitable value of λ. Through numerical experimentation, we find that the energy function used by Guerrero (et. al) does not give the desired results for CT image reconstruction, and that 2-norm regularization does not improve image quality. We therefore investigate using the Huber prior as a regularization term, as well as alternative energy functions. We use the Huber prior, and find a new energy function which is used to determine λ. Our modified hybrid approach is shown to significantly improve image quality in several numerical experiments.

Zechariah Hazel, Willamette University

Creating Problems: The Art of Stamping Graphs

The study of graph symmetries is an active inquiry in graph theory. Working from the basis of Albertson and Collins' paper on breaking graph symmetries by coloring vertices, this presentation will discuss a new way to distinguish graphs; by coloring the minimal k-connected subgraphs. Specifically, this presentation focuses on coloring 2-connected subgraphs (which we call stamps) of various families of graphs, and solving for the least number of stamps that must be colored to distinguish the graph. We present a solution to this problem for complete graphs and certain circulant graphs. Also, this presentation briefly discusses the parallel goal of our project; to develop meaningful connections between art and math, and demonstrate that the subjects are more similar than not.

Adeline Jacobsen, University of Washington Tacoma

Who's Gerry? Chasing The Math and The 'Mander

Gerrymandering is the manipulation of congressional districts to increase the probability of a desired outcome. In this presentation we will be chasing the math and the 'mander throughout history, starting from the very beginning of Gerrymandering back in 1812, to the various court cases and metrics that got us were we are today and ultimately to try and answer the question of "Who's Gerry?"

Ethan Jensen and Matthew Hull, George Fox University

SIMIODE Challenge Using Differential Equations Modeling

The Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations (SIMIODE) holds a Challenge Day using differential equations modeling (SCUDEM) every fall, hosted at local sites throughout the country. Three problems with concentrations in physics/engineering, life sciences/chemistry, and social sciences, are proposed and teams have a week to choose, develop, and analyze their model. We will present justifications for our assumptions in developing the model for our chosen problem as well as an analysis of the differential equation model and its application.

Samuel Lippert, University of Portland

Leavitt Path Algebras over Arbitrary Unital Rings and their Two-sided Ideals

We provide an analog of a Hilbert basis theorem for Leavitt path algebras (LPAs) over associative, unital, Noetherian rings. In addition we describe two classes of graded ideals of such rings, and provide a classification of all graded ideals using these classes. Lastly, we give characterization of when such LPAs are prime, and a description of graded prime ideals.

Lucas Perryman-Deskins, Willamette University

Sangaku in Spherical and Hyperbolic Geometries

During the Edo period (1603-1867 CE) Japan was almost completely closed off from the rest of the world and developed its own mathematical tradition called *wasan*. Part of this tradition was to hang tablets, known as *sangaku*, in the eaves of a shrine or temple as an act of devotion. The *sangaku* tablets were made of solid wood and each contained a colorful illustration of one or more results in Euclidean geometry. The tablets were created by people from many different social classes including farmers and samurai. In this presentation we will explore the generalization of some of these ancient Japanese theorems (or other more modern ones) in Euclidean geometry to both spherical and hyperbolic geometry. The basics of spherical and hyperbolic geometry will be explained. This research was conducted as part of the 2018 REU program at Grand Valley State University with Geneva Collins and under the mentorship of Professor William Dickinson.

Isabel Sacksteder, Willamette University

Modeling the Effect of Climate Change on Pine Snake Population

The sex of certain reptiles including the Pine Snake is impacted by the temperature during incubation. Researchers Burger and Zappalorti investigated this effect and found a linear relation between temperature and the male/female sex ratio. We can use this relation to model the effect of Climate Change on Pine Snake population dynamics. If we model a differential equation that defines the rate of change of the snake population, the solution to this differential equation will give us the population as a function of time. Analyzing this solution we can determine how long it will take for the sex ratio to cause Pine Snakes to go extinct. We will examine how this result changes for different temperature increase rates, and how we can incorporate evolutionary pressure and adaptation into our model.

Lingyu Wang, Willamette University

Investigating the Large-Scale Structure of the Universe by Calculating the Correlation Function and Power Spectrum

A density field, providing the matter density at positions in a regular three-dimensional grid, has recently been created from a catalog of galaxy clusters in the 2MASS redshift survey. This provides us with the opportunity to calculate the correlation function and power spectrum of the density fluctuation in the universe more accurately, since the density field characterizes the universe in a continuous way rather than discrete galaxy counts. We first calculate the two-point autocorrelation functions of the density map using python, where the BAO, a density fluctuation of baryonic matter, shows up as a bump in the autocorrelation function. Secondly, we apply the Fourier transform to the density field giving us an estimate of the power spectrum.

Breeann Wilson, Whitworth University

Capturing Disutility in the Classical Secretary Problem

The Secretary Problem is a model for sequential decision-making that has inspired many variations since its introduction over 50 years ago. In the classical setup of the problem, there are N scheduled interviews for a secretary position and an administrator must decide immediately after each interview whether to accept or reject that candidate. In the game, a rejected applicant cannot be reconsidered for the position and the interview process will end once a candidate is chosen. There is already a proven optimal strategy for hiring the best candidate in this classical setup where the sequence of candidate rankings is uniformly distributed in time and there is no expense associated with conducting the candidate interviews. However, these constraints are unrealistic if the administrator needs to hire someone quickly or if the interview process is costly to go through. In this talk, I introduce a variation of the classical setup that relaxes these constraints to more closely mimic a realistic interview process. This new model breaks the uniformity of the classical setup by introducing a weight that captures the disutility of conducting additional interviews and shows that imposing even infinitesimal costs on the interviews will force the probability of picking the best candidate to drop by nearly 10% when compared to the classical model.