

# OPTIX Module 3 – Advanced

## Spectroscopy of Rubidium

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### 1 Objectives:

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In this module you will learn about

- atomic energy levels, in particular the hyperfine structure of the alkali-metal atom rubidium;
- the correct use of an extended cavity diode lasers to probe atomic spectra;
- Doppler-broadened and Doppler-free absorption spectra of rubidium.

Note that you must take the [Module 3 - Intermediate](#) before you can take this module, since the intermediate version teaches you how to work with diode lasers in general and ECDLs in particular. Use this manual as you work through the module to keep track of your notes and thoughts. In addition, you may have to add a few printouts or add additional sheets of paper containing data tables, sketches, or additional notes. Please note that this is *not* your lab report and that you are expected to complete a full lab report written in *LaTeX* after you have completed this module. It should be a publication style report, which means that it should place a big emphasis on your data and data analysis, and not so much on all the nitty-gritty details of how to assemble the apparatus or your personal experience while completing this lab. Keep this in mind as you work through the module. Your instructor can provide you with more information and will send you a template file that you can use for your report.

### 2 Tests and assessment:

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In preparation for this module, read through the whole manual and answer the questions that are marked with a \*. You should also watch the [VIDEOS](#) that are posted on our website ([www.willamette.edu/cla/physics/info/NSF-OPTIX](http://www.willamette.edu/cla/physics/info/NSF-OPTIX)). They are meant to accompany this manual and will show you some critical steps of the module. When you come to lab, be prepared to discuss your answers to these questions with your classmates and your instructor.

In order to assess the success of this module, you will take a short assessment test before you start (“pre-assessment”), and another one after you have successfully completed this module (“post-assessment”). At this point you will also have the opportunity to provide us with feedback about the module that we will use to improve it for the next student generation. Thank you for your support!

### 3 Equipment:

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You can find most of the equipment for this module in the box labeled ‘Module 3’ that is located in one of the cabinets in the OPTIX lab. Standard equipment that is used in multiple modules is located in the cabinets in the OPTIX lab. Please feel free to also ask your instructor for help. You will need:

- A high voltage power supply with a Hydrogen-tube attached;
- yellow cardboard glasses with diffraction gratings (from Edmund Scientifics);
- a Thorlabs laser diode driver with current driver and temperature controller (LTC100-B);
- laser diodes centered at 780 nm (L780P010);
- a collimation lens for laser diode;
- a diffraction grating attached to a PZT and a mirror mount;
- silver coated mirrors and mirror mounts;
- the *Mightex* IR1 spectrometer;

- a saturated absorption glass cell containing rubidium;
- a solenoid that fits around the glass cell;
- a power supply for the solenoid;
- a small CCD camera;
- polarizing beamsplitter cubes;
- half-wave and quarter-wave plates for 780 nm light;
- a photodiode.

## 4 Required background knowledge and things you need to do before starting this module:

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Review the *Laser Safety Handout* before coming to lab. Make sure to always wear your goggles unless your instructor tells you that it is safe to take them off. You must have successfully completed [Module 3 - Intermediate](#) before starting this module.

Review your knowledge of atomic level structures from Modern Physics or another physics course. We will provide you with some facts, analogies, and “half-truths” in this module to help you develop a *conceptual* understanding of the energy level structure of rubidium, or at least give you a certain familiarity with it. There will be a few things you just have to believe at this point; you will see theoretical proof of them in our upper-level Quantum Mechanics course.

**Make sure to read through the *whole* manual before coming to the OPTIX lab, and mark everything that you find difficult to understand.** In addition, **work through all the boxes marked with a \***. **Be prepared to discuss your completed work with the group and your instructor before starting the lab. This counts as your pre-lab.** During your lab time, you can work through parts of the manual with your lab group and instructor, and you will get a lot of hands-on experience.

If you damage anything, please tell your instructor *immediately*. While we of course try to avoid damaging the equipment since it is rather pricey, especially in this lab, accidents can happen. Please don't try to cover up any damage; that only makes life harder for us as we try to figure out why something is no longer working. Tell us exactly what happened, and we can either fix it or replace the equipment quickly.

Lastly, write down everything related to atomic level structure you remember from Modern Physics or from another previous class, in the box below. And now - have fun in the lab!

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## 5 Introduction to atomic level structure:

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The idea that all matter is composed of small, indivisible objects is quite old: Already 440 B.C., Leukipp and his student Demokrit proposed the idea of an 'atomos', an indivisible object that creates all matter around us and determines its properties. By now we have lots of experimental proof that this idea is indeed correct. One of the earliest proofs was published in 1808 by Dalton. In his work 'A new system of chemical philosophy' he showed that the mass ratio of constituents before and after a chemical reaction remains the same. And more modern measurements like X-ray diffraction, Brownian Motion, or using Field Emission Microscopy have also again and again demonstrated the discrete nature of atoms. While classical and semi-classical models of the atom exist and lead to decent agreement between experiment and theory, quantum mechanics really is necessary to truly understand matter on the atomic level.

In the following sections we will guide you through those quantum mechanical models. We will start with the simple *Bohr Model* that already leads to a decent agreement with experimental observations as long as one doesn't look too carefully. You can think of the Bohr model as a modification of the Rutherford model in which the *quantization of the orbital angular momentum* of the electrons is included. In particular, we will look at the simplest atom, the hydrogen atom, and see how this changes its atomic level structure. Lastly, we will present, without proof, a few quantum mechanical findings and show how applying them to the hydrogen atom leads to the fine structure and the hyperfine structure. Think of all of these steps as observing the atomic level structure from far away (with low resolution), and then taking out your magnifying glass and zooming in more and more, thus allowing you to see the energy levels in more and more detail.

### 5.1 General properties of atoms, the *Rutherford Model*, and the *Bohr Model*.

Atoms are neutral. This means that the amount of positive and negative charges in each atom are exactly the same. In addition to these protons (the positive charges) and electrons (the negative charges), atoms also contain neutrons that are electrically neutral (therefore the name). In the simplest model of the atom, the *Rutherford Model*, protons and neutrons are clumped together in the *nucleus*, while the electrons move around the nucleus on *orbitals* a certain distance away from the nucleus. The analogy to the planetary system is very tempting, and while appropriate at this level, we will very soon see that it is completely incorrect. But for now, draw a sketch of the hydrogen atom in this simple model (note that it has only one proton and one electron).

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One way to experimentally confirm the neutrality of atoms is by measuring a vanishing force on an atom inside an electric field. Imagine placing an atom in between the plates of a parallel-plate capacitor, that - as you know - produces a very nice, homogeneous electric field. What would happen if the atom was not neutral?

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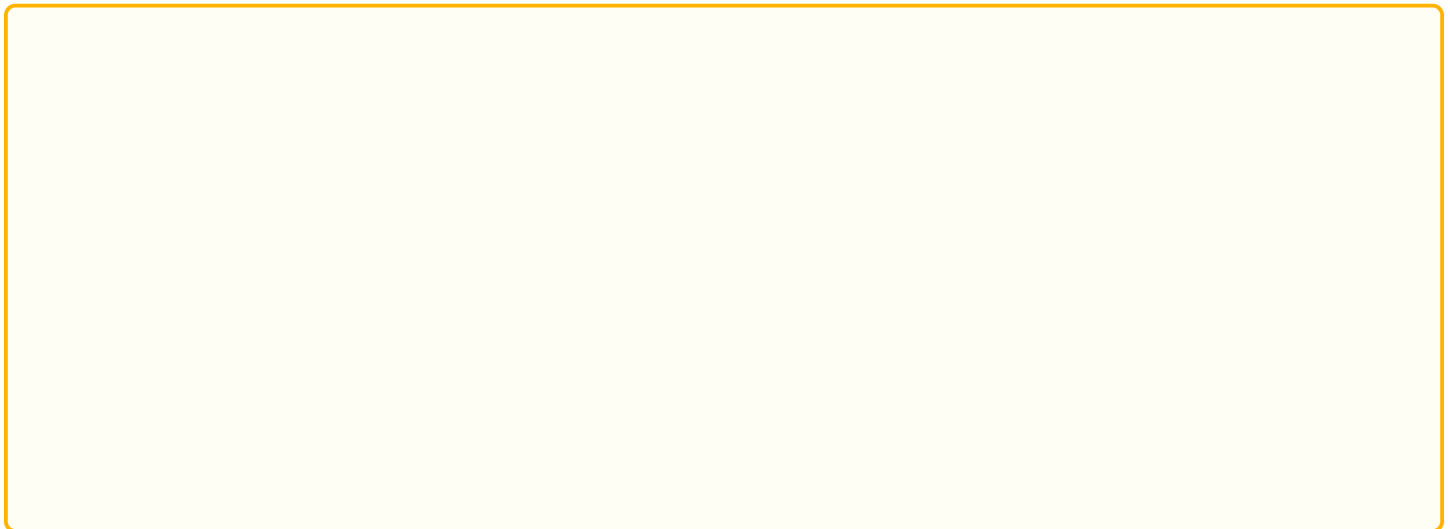
Can you think of anything else that could affect the motion of neutral atoms such that they *appear* to be non-neutral? In other words, how would you set up such an experiment and make sure that any effect you see is really coming from a non-neutral atom?

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Atoms that display similar properties are arranged in the same column of the *periodic table*. For instance, all atoms in the first column (the alkali-metal atoms) contain a single unpaired electron in the outer-most shell. Moving over one column to the right adds another electron to the atom, and - to keep the atom overall neutral - another proton as well. In many cases neutrons are also added. They essentially help glue the nucleus together and prevent the protons from flying apart as one would expect from equally charged particles in close proximity. Atoms of the same element can come with slightly different masses; these are called different *isotopes* of the same element. Isotopes have different amounts of neutrons in the nucleus. The amount of protons is the same, of course, and is what determines the physical and chemical properties of this particular element.

Atoms can absorb energy in the form of *photons*, or light packages, and this moves an electron from a lower energy state to a higher one. [Carefully re-read the previous sentence: It is the *atom* that absorbs the photon, but the *electron* that changes its energy state. Saying that the electron absorbs the photon is incorrect.] We say that the atom becomes *excited*. But just as a child excited to go to the playground cannot stay excited forever, atoms also have to “calm down” eventually: The excited electronic energy states are not stable, and the electron has to decay to the lowest possible energy state, the *ground state*, eventually. ‘Eventually’ typically is in fact a very small amount of time, on the order of  $10^{-8}$  seconds for most electronic states, but there are some excited electronic energy states in which an electron stays for a much longer time, up to several ms! These are the so-called *metastable* states. We already mentioned them in [Module 3 - Intermediate](#) when you learned about the three- and four-level systems that are used as a laser medium. Independent of the exact nature of the excited state, the electron eventually decays back into the ground state and emits another photon that carries away the additional energy. It also carries some momentum given by its wavelength ( $p = \hbar k = \frac{h}{\lambda}$ ), which means the atom experiences a small recoil every time it emits a photon. As an aside, this momentum transfer can be used to slow and trap atoms in a magneto-optical trap.

In a purely classical world, electrons could orbit the nucleus at pretty much any distance they like. But that would mean that the light that is emitted from atoms could span pretty much the whole spectrum without any preferences since electrons could move essentially freely from one distance to any other (smaller) distance. Experimentally, however, we find that is not the case. The light emitted from atoms is very much discrete. Convince yourself that this is indeed true. Take the high-voltage power supply out of the cabinet (it should be located on the same shelf as the box labeled ‘Module 3’) and make sure that the glass tube that is held between the springs in this supply is labeled as hydrogen. Turn on the supply, and you should hear a humming and see the glass tube light up. Careful: Always make sure that the glass tube is safety attached to the power supply before turning it on, and never turn it on without the tube present. The high voltage could seriously injure you if you touched the metal leads with your hands! Put on the yellow cardboard glasses that contain diffraction gratings similar to the ones you encountered in [Module 1](#). This will allow you to see the spectrum of hydrogen. You might want to turn off the room lights for this observation. Describe what you see and add a sketch.



The fact that there are only a few discrete emission lines tells us that there can only be a few energy levels that are *allowed*, i.e. accessible to the electrons. How else could there be only so few lines? Quantum mechanically, we can easily understand that. Even the word ‘quantum mechanics’ already tells us that it deals with things that are ‘quantized’, i.e. come in discrete packages. So this would mean the end of Rutherford’s model of the atom, wouldn’t it? After all it made no predictions and has no explanations for this discrete behavior. Ah, but wait, we can fix that! In order to understand this observation (almost) classically, all we have to do is require that the electron’s *orbital angular momentum* is quantized! This may come out of nowhere, so let’s think about it for a moment to see if we can make sense of it.

Classically, the electron’s orbital angular momentum  $\vec{L}$  determines how quickly the electron moves around the nucleus at a given distance  $r$ . Remember from Intro Physics I that  $\vec{L} = \vec{r} \times \vec{p}$ , where  $\vec{r}$  is the vector pointing from the origin to the object (here, from the nucleus to the electron), and  $\vec{p} = m\vec{v}$  is the (linear) momentum of the object (here, the electron). For a fixed distance  $r$  from the nucleus, the orbital angular momentum only depends on the (linear) momentum. Let’s for now accept the

statement that the orbital angular momentum is quantized and see what the implications of this assumption are. Let's also for now assume that  $r$  is fixed. Toward the end of this discussion we will revisit the distance  $r$  and see if it can be arbitrary, as predicted by the Rutherford model of the atom, or if one of the implications is that the distance is quantized (as we know is true from observations).

At a fixed distance  $r$  from the nucleus, a quantized orbital angular momentum also means a quantized linear momentum. Now, to understand *why* it makes sense for the linear momentum to be quantized we have to introduce another strange concept from quantum mechanics: The fact that particles such as the electron are actually also a wave (you know this from Modern Physics). The wavelength of these waves is given by the *deBroglie wavelength*  $\lambda = h/p$ , where  $h$  is the Planck constant. Thus, an electron with a given orbital angular momentum at a given distance from the nucleus has a given wavelength. Derive an expression for the orbital angular momentum of the electron in terms of this wavelength using the expressions introduced so far. Hint: Think about the relative orientation of  $\vec{r}$  and  $\vec{p}$  for an electron on a spherical orbit.

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Now imagine the electron could have an arbitrary momentum (and thus an arbitrary wavelength), and picture the electron wave as it moves around the nucleus. This wave will most likely look very different after a full trip around the orbit, and it will look different yet again after a second full trip. For example, if at some point on the orbit you had a crest after the first full trip around the orbit, you may have a trough there after two trips, or anything in between! In more technical terms, the *phase* of this electron wave is different after zero, one, two, etc., full trips around the orbit, and it is not equal to a multiple of  $2\pi$ , which would lead to waves that look identical. All of these differently looking waves overlap in space because the electron stays at a fixed distance as it orbits the nucleus. But what happens when you overlap many waves that have different, arbitrary phases?

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Let's play with some math to show that your intuitive answer to the last question agrees with mathematical predictions. Then we'll think more deeply about the consequences of these arbitrary phases.

### Let's play with it!

Remember that we can describe sinusoidal waves by  $y(x, t) = A \cos(kx - \omega t + \phi_0)$ , where  $k = \frac{2\pi}{\lambda}$  is the wave number,  $\omega = 2\pi f$  the angular frequency,  $A$  the amplitude of the wave,  $\phi_0$  the initial phase, and  $\lambda$  and  $f$  are related through the speed of light,  $c = \lambda f$ . To simplify things, let's assume that  $A = 1$  and  $\phi_0 = 0$ . Our initial wave at  $t = 0$  is then simply given by  $y(x, 0) = \cos(kx)$ . Let's further say that a full trip around the orbit takes a time  $T$ . So, after one full trip the wave is then given by  $y(x, T) = \cos(kx - \omega T)$ . Since we are assuming a completely random wavelength and an unrelated time  $T$ , let's just pick some random numbers to see how the wave changes after one round trip, for example let  $k = 1 \text{ m}^{-1}$  and  $\omega T = 1$ . That simplifies things to  $y(x, T) = \cos(x - 1)$ . Using the same random numbers for  $k$  and  $\omega T$ , derive a similar expression for  $y(x, 2T)$ ,  $y(x, 3T)$ ,  $y(x, 4T)$ , and  $y(x, 5T)$ .

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Then write a simple Python program that plots these five waves as a function of  $x$ . The figure on the next page might help you get started. Also plot the sum of these five waves. Attach a printout to this manual. Does the plot confirm your intuitive prediction?

```
sines.py - C:/Users/Labrats/Dropbox/Grants/NSF_IAP_2014/Teaching Modules/Module 3/sines.py (3.5.1)
File Edit Format Run Options Window Help
# First we need to import the correct modules. We'll do this in two different ways:
# The first one is the one that allows us to plot. We'll import the full module.
# What that means is that we have to refer back to it when we use functions in that module.
# That's why we give it the nick name 'plt'.
import matplotlib.pyplot as plt

# From the second module we only import three elements:
# The sine function, pi, and 'arange' which is a built-in function that allows you to
# determine 'a range' of numbers.
from numpy import sin, pi, arange

# Now we use arange to define x from 0 to 2 pi in steps of 0.01 pi.
x = arange(0.0,2*pi,0.01*pi)
```

Remember that we are describing the electron by such a superposition of waves that all have different, random phases. But as you just saw, after many round-trips the superposition of these waves vanishes, i.e. the wave interferes destructively with itself! But that would mean that the electron vanishes, too! That can't be good. The only way to prevent this from happening is to only allow certain wavelengths, those that form *standing waves* along the orbit. Let's briefly review standing waves before we move on: Think for example of string instruments. The reason we hear a certain pitch from a certain string is because its length and tension support standing waves of the wavelength corresponding to that pitch (and its higher harmonics). Draw the first three standing waves on a string of length  $L$  in the box below.

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Similarly, if the electron wave forms a standing wave along its orbit, then the waves after zero, one, two, etc., full round-trips look exactly the same, they have the exact same phase (modulo a multiple of  $2\pi$ ), and so all of these waves interfere *constructively*. Derive an expression for the wavelength of such a standing wave a distance  $r$  from the nucleus. Also derive an expression for  $k$  and  $\omega T$  and show that they must be related. Lastly, derive an expression for the quantized orbital angular momentum  $L$ .

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Hopefully you now understand why it makes sense that the (linear) momentum and thus the orbital angular momentum are both quantized: Because the wavelength of this electron wave can only have certain discrete values. As mentioned earlier, we should now check if the radius of the orbit, which we so far have only required to be constant, can have an arbitrary value, or if it is also quantized. Since we assume the electron to be in circular motion around a charged nucleus, we have a balance of forces:  $\frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$ . Explain this relation.

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Lastly, use the expression you derived for the quantize orbital angular momentum together with  $L = mvr$  (which is correct for circular motion), solve it for  $v$ , plug this expression into the balance of forces, and finally, solve that relation for  $r$ . As you will find, the result indeed tells you that there are only discrete distances from the nucleus that are accessible to the electron, because only these distances lead to standing waves.

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Note that if you define the *Bohr Radius* as  $a_B = \frac{4\pi\epsilon_0\hbar^2}{me^2}$ , your final result should simplify to  $r_n = n^2 a_B$ . Or in other words: There is a smallest allowed orbit of radius  $a_B$ . All excited orbits ('states') have distances from the nucleus that are 4, 9, 16, etc., times larger than this ground state orbit. Given the simplicity of the arguments we just used, it is actually quite amazing how well this result agrees with experimental data, at least if we don't look too closely. This modified version of the Rutherford model is called the *Bohr model of the atom*.

Before we move on, go back through this section and work out the most important steps. What was our assumption and how did this assumption lead us to this final result? Lastly, sketch the allowed orbits.

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## 5.2 Orbital angular momentum and the hydrogen atom

The hydrogen atom is the simplest atom, consisting of only one proton and one electron (and no neutrons). Using the Bohr model, especially our result  $r_n = n^2 a_B$ , together with the potential energy between two charged particles,  $U = \frac{q^2}{4\pi\epsilon_0 r}$ , we can derive an expression for the allowed discrete energy values. Do this and sketch them. Comment on the behavior as  $n$  gets large and explain what a large  $n$  correspond to. [Hint: The *total* energy must include the kinetic energy!]

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Note that the energies are negative, indicating *bound* states, and thus the largest possible energy is equal to zero as you hopefully just found in the previous box. The electron can transition between these bound states. Derive an expression for the frequency of such a transition from a state with energy  $E_{n_1}$  to a state with energy  $E_{n_2}$ . Then convert your result into an expression for the wavelength of such a transition. Lastly, find numerical values for the transitions  $(n_2 = 2, n_1 = 1)$  and  $(n_2 = 3, n_1 = 2)$ , and compare them to the spectrum you observed earlier.

Because this is a very simple model, the allowed energy states do not fully agree with experimental observations. They are quite close, but as you look more and more carefully (as you zoom in more and more), you start noticing differences. The reason for these deviations is that even the simple hydrogen atom is, of course, not a classical object. Both the electron and the proton carry additional quantum mechanical properties, for example, they both have a property known as ‘spin’. Classically, we can picture a spinning sphere, and the spin then characterizes how quickly the sphere spins around. Of course, since the electron is a point-like particle and not an extended object, it is in fact quite difficult to envision it spinning. So keep in mind that the ‘spin’ is really something totally different, it is a quantum mechanical property of the electron that has no classical equivalent. Let’s review this spin a bit more since we’ll need it and other ‘spin-like’ quantities to make sense of the absorption spectrum of rubidium.



### 5.3 Quantum mechanical spins and how to couple them

Quantum mechanically, a generic spin  $\vec{A}$  is a vector of length  $\sqrt{A(A+1)}\hbar$ . If there is no preferred direction in the problem you are considering, this vector can be aligned in any arbitrary way. In many cases, though, there is a *quantization axis* that breaks the symmetry of the problem. For example, picture a perfect sphere. This sphere has no preferred direction, but if you imagine rotating the sphere about a rotation axis you break the symmetry. Very often, the direction of such a symmetry breaking axis is called the *z-direction*. For atoms, such a symmetry breaking element is typically either an electric or a magnetic field, and the direction of this field then coincides with the quantization or *z*-axis. When such a quantization axis is present, a spin can only have certain discrete orientations with respect to it such that the projected length onto this axis is given by  $A_z = m_A\hbar$ , where  $-A \leq m_A \leq A$ , and  $m_A$  can only change in steps of one. Therefore, there are  $2A + 1$  values for  $m_A$ , and thus  $2A + 1$  projections of the vector  $\vec{A}$  onto the quantization axis. Sketch this for  $A = 1/2$  and  $A = 2$ .

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To understand the energy level structure of rubidium we need three spin-like quantities: (a) the electron spin  $\vec{s}$ , (b) the electron orbital angular momentum  $\vec{l}$ , and (c) the nuclear spin  $\vec{I}$ . As you probably remember, the electron spin  $\vec{s}$  has a length  $|\vec{s}| = \sqrt{s(s+1)}\hbar = \sqrt{\frac{1}{2}(\frac{1}{2}+1)}\hbar = \sqrt{\frac{3}{4}}\hbar$  and two possible projections because  $s = \frac{1}{2}$ , and thus  $2s + 1 = 1 + 1 = 2$ . These projections are  $s_z = m_s\hbar = \pm\frac{1}{2}\hbar$ , and they are also often referred to as the ‘up’ or ‘down’ state. *Pauli’s exclusion principle* tells us that two electrons can only share the same electronic state if their spins point in opposite direction, which means that one is in the ‘up’ and the other in the ‘down’ direction. This also explains why the first electron orbit can only hold two electrons; trying to squeeze a third electron into it would violate Pauli’s principle, and thus this third electron must move to a higher energy level state.

The orbital angular momentum  $l$  can have any value between 0 and  $n - 1$ , and we know that its length is given by  $|\vec{l}| = \sqrt{l(l+1)}\hbar$ , and its projection by  $l_z = m_l\hbar$ , where  $-l \leq m_l \leq l$ , and  $m_l$  changes in steps of one. For example, an  $n = 1$  state has only  $l = 0$ , while an  $n = 2$  state can have either  $l = 0$  or  $l = 1$ . When the spectral lines were first observed, they were cataloged using capitalized letters from the alphabet. This system is still in use, but now we know that the ‘S’ state is really a state that has  $l = 0$ , the ‘P’ state has  $l = 1$ , and the ‘D’ state has  $l = 2$ . From there, the nomenclature follows the alphabet:  $l = 3, 4, 5, \dots$  are called ‘E’, ‘F’, ‘G’ states, and so on.

The nucleus, too, carries a spin, the nuclear spin  $I$ . In the case of the hydrogen atom it is also equal to  $\frac{1}{2}$  because the nucleus of a hydrogen atom consists only of a single proton, and the proton spin is  $\frac{1}{2}$ , just like the electron spin. For atoms with more protons and neutrons in the nucleus the situation is a bit more complicated since each proton and neutron contributes a spin of  $\frac{1}{2}$ , and the nuclear spin  $I$  is a combination of all these individual spins. For the two rubidium isotopes you will be working with, the nuclear spins are  $I = \frac{5}{2}$  ( $^{85}\text{Rb}$ ) and  $I = \frac{3}{2}$  ( $^{87}\text{Rb}$ ).

Now we get to the interesting part: All of these projections are completely degenerate in the Bohr model, which means that they all have the exact same energy. But quantum mechanically, these different spin-like quantities all “know of each other’s presence”, which means they *couple* to form a new vector, a new spin-like quantity. This coupling lifts the energy degeneracy; when we look really carefully, we see that the Bohr orbits don’t just have a single allowed distance from the nucleus, but rather have several closely spaced energy levels at slightly different distances from the nucleus.

In alkali-metal and other light atoms, the first coupling, the one corresponding to the largest energy splitting, is given by the coupling of the orbital angular momentum  $\vec{l}$  and the electron spin  $\vec{s}$ . Even though this coupling has the biggest effect on the energy splitting, it is still an overall very small effect and is thus called the *finestructure* of the atom. Note that there are also other effects that contribute to the finestructure, like relativistic effects. These affect the exact *shifts* in energy, but don’t change the overall *conceptual* narrative (much). We will thus ignore them here and refer you to our upper-level Quantum Mechanics course if you’d like to learn more. The quantum mechanical symbol for the finestructure is  $\vec{j}$ , and it is given by the vector sum of  $\vec{l}$  and  $\vec{s}$ :  $\vec{j} = \vec{l} + \vec{s}$ . Just like every quantum mechanical vector or spin-like quantity, its length is given by  $|\vec{j}| = \sqrt{j(j+1)}\hbar$ , and its projection is  $j_z = m_j\hbar$ , where  $-j \leq m_j \leq j$ , and  $m_j$  can change in steps of one. We can further simplify this result since  $s = \pm\frac{1}{2}$  only. Insert this and derive expressions for the possible  $j$ -values in the rubidium atom (there’s a box on the next page).

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Just as the naming of the different  $l$  states follows historic roots, so does the notation for the overall energy levels: They are typically labeled as  $n^{2s+1}l_j$ . For example, a state with  $n = 4$ ,  $l = 2$ , and  $j = \frac{5}{2}$  is called a  $4^2D_{\frac{5}{2}}$  state.  $2s + 1$  is also called the (spin) *multiplicity* of the state since it counts how many finestructure components there are (convince yourself that this is true for the above example). Note that according to our rule  $0 \leq l \leq n - 1$ ,  $n = 1$  can only support an  $l = 0$  or S-state. For  $n = 2$ , we can generate one S-State ( $2^2S_{1/2}$ ) and two P-states ( $2^2P_{3/2}$  and  $2^2P_{1/2}$ ), and so on.

In addition, the nuclear spin  $\vec{I}$  also couples with the orbital angular momentum and the electron spin. For light atoms,  $\vec{I}$  couples with  $\vec{j}$  to form  $\vec{F} = \vec{j} + \vec{I}$ . This is called the *hyperfine structure* because the associated energy shift is even smaller than the one from the finestructure. Just like all other quantum mechanical vector quantities, its length is given by  $|\vec{F}| = \sqrt{F(F+1)}\hbar$ , and its projections are  $F_z = m_F\hbar$ , where  $-F \leq m_F \leq F$ , and  $m_F$  changes in steps of one. Note that for heavier atoms, this strict “ $\vec{s}$  and  $\vec{l}$  couple first, then the new vector  $\vec{j}$  couples with  $\vec{I}$ ” rule is no longer strictly obeyed, and the resulting spectra get more complicated.

That was a lot of theory! Make sure you understand the big picture, and ask your instructor or your group members if you don't.

## 5.4 The hyperfine structure of rubidium

If you think you understand the gist of things, try to apply what we just learned to rubidium. In the following box, start on the left by sketching out the energy levels according to the Bohr Model. Do this for both isotopes,  $^{85}\text{Rb}$  and  $^{87}\text{Rb}$ , and for  $l=0$  and 1. Then include the finestructure, and lastly the hyperfine structure. Don't worry about the exact energy spacings; feel free to grossly exaggerate the energy shifts from fine- and hyperfine structure. For rubidium, the levels with larger  $j$  or  $F$  have higher energy than the levels with smaller  $j$  or  $F$ . Make sure to label every state correctly.

## 6 The experimental setup:

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Review or complete [Module 3 - Intermediate](#) before starting this module. You need a working ECDL. It is up to you whether this ECDL is in Littrow or in Littman-Metcalf configuration. Check that you can tune it over several nanometers without losing feedback and without getting lasing on multiple modes.

## 7 Using the ECDL to study atomic rubidium:

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We will now use this working ECDL to study the atomic energy levels of rubidium. In the box labeled ‘Module 3’ you will find a glass cell that contains rubidium vapor (it’s in the metal cylinder; please handle it with care and don’t touch the ‘caps’ of the cylinder with your bare hands). Carefully mount it using clamps and place it in the path of the laser beam such that the beam goes through the center of the cell (along the cylinder axis). **Be careful to avoid reflections off the cell back into the laser (since that could easily damage the laser diode)** by slightly tilting the cell just a little with respect to the laser beam, such that any reflections off the cell’s surfaces will not make it back into the laser diode. Place the small webcam next to the cell such that you can see the full length of the cell on the computer screen. You should see two bright spots, one on the entrance and one on the exit window; that’s where the laser beam hits the cell. Place the spectrometer such that you see the spectrum of the laser diode (for instructions how to operate it refer back to [Module 1](#) and watch the [VIDEO](#) called [How to use the Mightex Spectrometer](#)). Use the internet to find the correct wavelength to excite the  $5S_{1/2}$  to  $5P_{3/2}$  transition in rubidium and record it below. Note: Search for ‘D Line Data’ to find a highly accurate value for this wavelength, and record the wavelength for both isotopes to the precision given in the D Line Data file.

### 7.1 General setup:

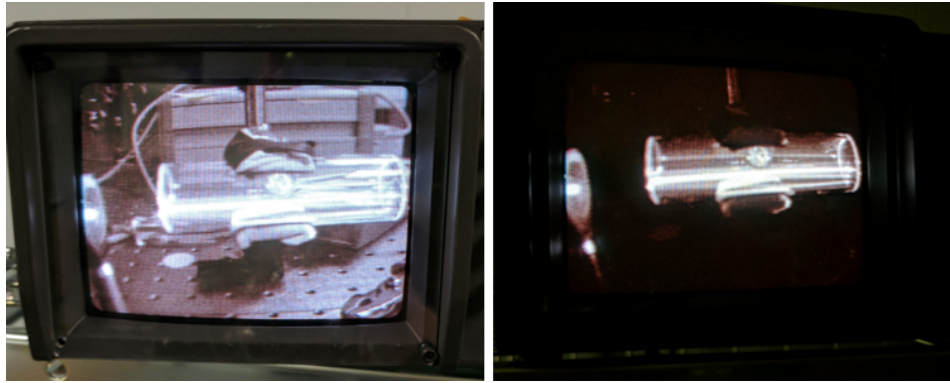
With the up/down screw in the middle of the feedback range, rotate the left/right thumb screw to pull the ECDL to that wavelength, and confirm that you can cover a range of about  $\pm 1$  nm around that wavelength without any mode hops or multi-mode behavior. If the current wavelength is too far away and you cannot pull the ECDL to the desired wavelength with the screw alone, what else can you change to affect its wavelength? Record all changes and any new or updated settings in the box below.

Once you know that you can tune the laser over the desired wavelength look back at the camera feed on the screen. To make the following a bit easier to see, let’s use the piezo-electric transducer (PZT) that is glued between the diffraction grating and the mirror mount. The PZT is a piece of ceramic that changes its length slightly when you apply a voltage to it. We will use a function generator to apply a periodically changing voltage to the PZT. Find one (it should be in one of the upper cabinets), set it to the saw tooth function at a frequency of about 200 to 300 Hz, and connect it to an oscilloscope (located in the same cabinet) to check the signal. Use a BNC cable for this and add a T-connector at the oscilloscope to split the signal (the second signal will go to the PZT after you have confirmed that the signal looks good). **Note: If you have problems using function generators or oscilloscopes please watch these introductions: [www.youtube.com/watch?v=Zink6v6TXk4](http://www.youtube.com/watch?v=Zink6v6TXk4) and [www.youtube.com/watch?v=dBVWv7enDsU](http://www.youtube.com/watch?v=dBVWv7enDsU), ask your instructor, or work through [Module 1 - basic \(b\)](#).**

After you confirm that the signal is indeed a saw tooth with a frequency of about 200 Hz, connect a second BNC cable to the T and connect the other end of this BNC cable to the PZT. This signal now periodically changes the voltage and thus the

length of the PZT, which in turn changes the length of the extended cavity, and that in turn changes the frequency of the laser with the same period. In other words, connecting a function generator to the PZT allows us to *scan* the frequency of the laser about the center frequency that is given by the position of the left/right thumb screw and over a range that is given by the amplitude of the saw tooth.

With the PZT scanning the length of the extended cavity, observe the camera feed as you slowly rotate the left/right thumb screw on the grating mount. You want to cover a range from just below to just above the resonance wavelength of rubidium, so observe the spectrometer as you do this to make sure you are not moving off too far. When you hit the right wavelength and are in resonance with the atoms you should see a bright line appear along the cylinder axis of the glass cell ('fluorescence') as shown in the photos below (the right photo is taken with the room lights off).



Find this line, then carefully tweak *both* thumb screws (left/right *and* up/down) to maximize the brightness and stability of this line. You can also adjust the current slightly. It is normal for a diode laser to slowly drift out of resonance over the course of a few minutes, thus reducing the brightness of the fluorescence in the glass cell slowly. You want to make sure that you don't see any fast blinking of the bright line, or any jumping away from it. In the box below, sketch what you saw on the computer screen and explain carefully and in detail what causes this bright line.

You know you are close to the atomic resonance when you see a stable and bright fluorescence signal, because that means that the atoms are scattering a lot of photons. But the spectrometer or even the fluorescence alone are not sensitive enough to tell us whether the laser is really lasing on a single mode or if multiple modes are contributing to the fluorescence signal (this is actually very likely!). In order to make that call we need an atomic *absorption* signal. To generate it, we will place a photodiode (PD) behind the saturated absorption cell and measure the intensity of the laser after it has passed through the cell. Sketch the setup in the box on the next page. Then predict what the photodiode signal should look like as you scan from below the atomic resonance across the resonance and finally to above the resonance. Relate the expected absorption signal to the observed fluorescence signal.

Note: In the following you will use an oscilloscope, a function generator, and a multimeter quite a bit. Make sure that you are comfortable with all of them. To refresh your memory, look over [MODULE 1 - BASIC \(Lab Equipment\)](#) and/or follow the links in that module.

The PD you will use is very sensitive. So before you place it into the laser beam, adjust its sensitivity using the rotating switch on the side such that it is set to the lowest sensitivity setting. This avoids saturating and possibly destroying the PD. Consult its manual (you can find it in the box labeled ‘Module 3’) for additional help on how to use it correctly. Use a multimeter to measure the voltage coming out of the PD by placing one lead into the center of the BNC pin and pressing the other against the outside of the BNC pin (this is ground). If this voltage is close to plus or minus 15 V you are saturating the PD and you need to attenuate the laser beam further. You can do this either by inserting a glass slide and using the weak reflected beam, or by inserting a neutral density filter after the saturated absorption cell. When the PD signal is on the order of 5 V or so, connect it to channel 2 of the oscilloscope using a BNC cable. Take a “T”-connector and split the signal from the function generator to send that signal both to the PZT and to channel 1 of the oscilloscope. We are now ready to look at the absorption signal.

When you first turn on the oscilloscope, chances are it is set to the *time mode* (you can check this under the ‘display’ options). That means that the vertical axis displays the voltage and the horizontal axis corresponds to time. You can adjust the resolution of both axes by rotating the corresponding knobs. The screen will display a number that tells you how much voltage or how much time one division on the screen corresponds to. If, for example, the number is 1 ms, then each division, i.e. each square, has a length of 1 ms, and the full length that is displayed on the screen corresponds to 10 ms. Turn on both channels. Chances are, you will not see a stable signal, but rather lines that move across the screen more or less erratically. That’s because the signal is displayed at a rate that is given by the time scale that you chose for the horizontal (time) axis. For example, if you chose 1 s per division, then the signal refreshes every 10 seconds. But periodic signals like the absorption signal have their own refresh rates (in this case it is given by the frequency of the function generator) that rarely coincide with the oscilloscope’s rate, leading to these jumpy signals you see on the screen. We can stabilize the signal by setting the *trigger* correctly. The trigger tells the oscilloscope when to refresh the screen. And if we sync the refreshing rate of the screen with the periodic signal we want to measure, then the display seems to be frozen and stable. This is similar to flashing a strobe light in sync with a jumping person. If you turn on the light every time the person is at their highest point, the person seems to be frozen and hovering in air! Let’s quickly review how to set the trigger correctly.

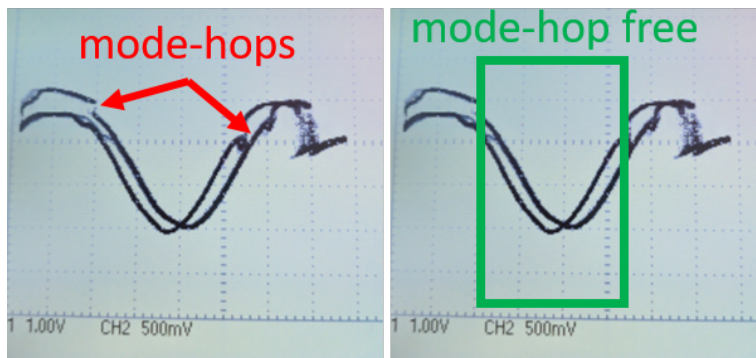
#### How to use an oscilloscope: The trigger:

1. **Determine the signal that will provide you with a trigger:** Quite often, this is the stable periodic signal coming out of a function generator (which you have connected to channel 1).
2. **Determine whether you want to trigger on a rising or a falling slope:** If you trigger on a rising slope at a level of e.g. 3 V, then the oscilloscope waits until it detects a voltage that comes from values smaller than 3 V, crosses 3 V, and goes to values larger than 3 V, and it will refresh the screen every time it detects such a signal. Similarly, if you trigger on a falling slope, the oscilloscope will look for a signal that drops through 3 V.
3. **Set the trigger level:** The trigger level should be set to some voltage value between the minimum and the maximum amplitude of your trigger signal.

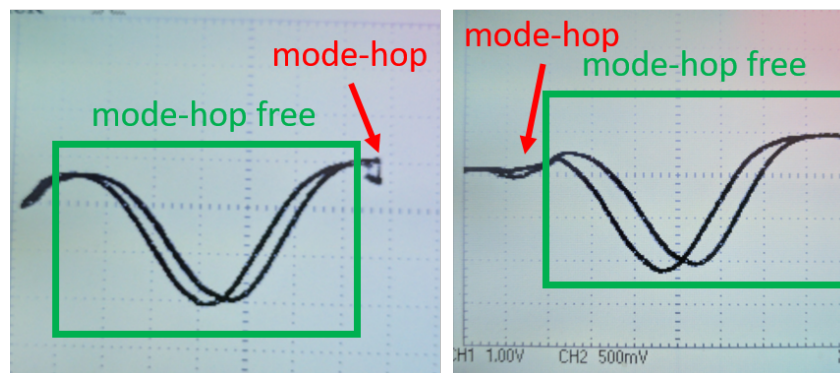
Discuss with your group whether you should trigger on channel 1 or 2 and what the trigger level should be. Also discuss whether triggering on a rising or a falling slope makes a difference (observe experimentally what changes when you change from a rising to a falling slope and describe and/or sketch it below).



With the trigger set correctly, observe the absorption signal on the oscilloscope and compare it to your predictions. Chances are, your absorption signal is not yet optimized and doesn't look quite as nice as you had expected. For one thing, you probably see mode-hops (sharp kinks, jumps, jagged lines, or very "fuzzy" regions). The photo below shows you an example of such a poorly optimized signal. Note that there are mode-hops on both sides of the signal, and that the mode-hop free region is pretty small.



Carefully tweak the up/down and left/right screws on the grating mount to optimize the signal until the absorption profile on the oscilloscope is clearly visible and stable over the whole range of interest (usually that covers several hundred to 1000 MHz or the full absorption profile). An example of a pretty good signal is shown in the photos below.



Don't worry if the signal jumps away and disappears while you touch the screws; that's normal because you are changing the length of the cavity slightly simply by touching the mounts. As you rotate one of the screws in one direction, you should see the profile disappear, reappear again, disappear again, reappear again, and so on, until you've rotated the screw too far and

the laser diode doesn't see feedback anymore. Observe the absorption profile after each of these tiny rotations and decide whether it looks better or worse (i.e. maximize the mode-hop free region), and adjust the screw accordingly. When you think you have a stable signal, move the PZT offset on the function generator to shift the whole profile left and right. The profile should shift *as a whole*; it should not change in shape or get distorted, and it should not jump away. Make sure that no mode-hops are "hiding" in the profile by changing the current over a few mA. Again, the whole profile should move to the left or right and should not change its shape or get distorted. If you notice some "fuzziness" appear as you change the current - that's a mode-hop (you can see one in the bottom-left photo on the previous page, on the right edge of the profile). It is OK to have mode-hops to the far left and far right of the profile we are interested in, but make sure that the actual relevant part of the profile (basically the whole dip) is mode-hop free. If a mode-hop is present, re-optimize the grating and - if necessary - find another temperature/current range that provides a more stable signal. **Take your time optimizing this signal; it takes some practice and patience.** Please check with your instructor when you think you have a good, stable absorption profile. Record any observations, concerns, or strategies in the box below.

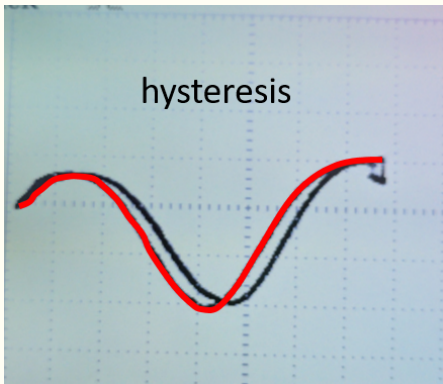


Before we move on, let me show you another way to display the absorption signal that makes it a little prettier (in fact, the photos you saw on the previous page were taken this way). Go back to the 'display' options and select 'xy-mode'. In this mode, the oscilloscope displays the voltage on the second channel as a function of the voltage on the first channel. What does that mean in our case? What is displayed as a function of what? Carefully think about the *physical* quantity is displayed on the horizontal axis!



In this setting, you will notice that you actually see a double absorption profile (see the photo on the next page (color online))! This is just an artifact of how we generate this signal: The PZT has *hysteresis*, meaning that its length at a certain voltage depends on whether you reach that voltage coming from lower or from higher voltages, and the two lengths are slightly different. So, one of the profiles you see on the oscilloscope in xy-mode is when the PZT expands (the red trace in the photo on the next page), and the other when the PZT shrinks (the black trace in the same photo). Explain why that leads to two nearly (but not quite) overlapping absorption profiles, and how they relate to the profile you saw previously when you

displayed voltage-vs-time.



When you have a good and stable profile, record the exact current and temperature values below.

You will now learn how to carefully change the wavelength of the laser while avoiding mode-hops, which allows you to see multiple absorption dips. Carefully tweak the offset on the function generator while at the same time slightly adjusting the current whenever you see a mode-hop appear. Slightly tweaking the current pushed the mode-hop back. You should see two absorption dips right next to each other, and by adjusting the offset and current in sync you can move one or the other into the center of the oscilloscope screen (and make it completely mode-hop free). Practice this a few times and note the direction in which you move the offset (clockwise or counterclockwise) and the direction you change the current (to lower or higher values) to move from the smaller of the two dips to the larger of the two dips. Record these directions in the box below.

We will think about where these two dips come from in just a moment. But there are actually two *additional* dips located further to the side of the larger of the two dips you currently see. To reach them, continue tweaking the offset and adjusting the current to move the spectrum over (check the directions you just recorded in the previous box). Do this very slowly and carefully; chances are you will run into several mode-hops. Adjust the current carefully to keep pushing the mode-hop to the side, off the screen. Keep going and, if you did it correctly and did not jump onto a different mode, you should see first one additional smaller dip appear, and then another, even smaller one. If you can't find them, go back to your starting values (that's why you recorded the temperature and current above) and try again.

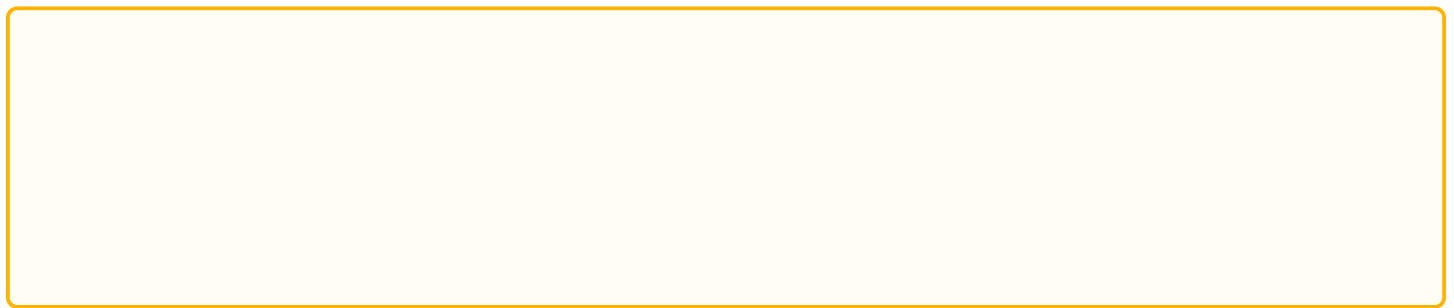
So why are there four dips, which atomic transitions do they correspond to? To answer these questions, check your sketch of the atomic level structure on page 10 of this manual, or consult the D Line Data pdf. (Note: There is a box on the next page.)





## 7.2 Data recording and analysis:

Ask your instructor for the 4-channel oscilloscope that can record the traces and save them to a memory stick. Then record a trace each for all of the four dips you just saw by centering them first, checking that you can see the full absorption profile without any mode-hops, inserting a memory stick, and then pressing the 'print' button on the oscilloscope. The oscilloscope saves the files in a folder called 'ALL00xx', where 'xx' is some number. Record this number together with a brief comment that identifies which of the four individual dips you just recorded. The actual data file is the .csv file within those folders.



To analyze these data files, we will now read the traces into a Python program to fit them with a Gaussian function, which will allow us to extract the approximate temperature of the gas! But before you go ahead and write the program, let's think about the physics some more, because we have a *small* problem: The traces you just recorded show the absorption in *volts* and the horizontal axis in *seconds* (or fractions thereof). Physically, we know that the absorption should be a fraction (somewhere between 0 and 100), and that the horizontal axis is proportional to the frequency of the laser. Let's tackle these problems one at a time. How can you convert the vertical axis from volts to a percentage (box on the next page)?

Converting the horizontal axis from time to frequency is a bit more tricky, but the two large dips that are close to one another can help us. Adjust the spectrum on the oscilloscope such that you *see the minima of both dips without any mode-hops*. Save a trace (record the number in the box below). The frequency difference between these two minima is actually very well known. Use your conclusions from previous boxes, in particular the ones in which you identify the four dips, and the information provided in the D Line Data sheets to find a value for this frequency difference. Briefly describe what you did, show your work, and record the final value below. A few hints: (1) You will need *both* pdfs for rubidium to answer this question. (2) The pdfs show you the hyperfine structure. You may assume that the strongest transition, i.e. the transition that corresponds to the maximum in absorption, is the *stretched state* (from the upper hyperfine ground state to the upper-most hyperfine excited state). Check with your instructor before moving on.

We will now fit the last data set you just took (which should still be in the format voltage-vs-time) to extract a value for this time difference. [Note that depending on how well aligned the ECDL is, you may be able to see both Gaussian absorption dips quite well, or you may have mode-hops to either side of the two minima. The latter situation may make the fitting procedure difficult. Please check with your instructor before moving on.] By knowing that this time difference corresponds to the frequency difference you just calculated, we can find the *calibration factor* that will allow us in a moment to convert the widths of the absorption dips into actual frequency widths (the full-width half-max of the Gaussian). To do this, let's write a Python program:

Open *Spyder* or any other Python editor on the lab computer. First, we need to import the packages we'll use. Please copy the following text into Python (without the bullet points):

- `import matplotlib.pyplot as plt` (This allows us to make nice plots of the data before we try to fit it.)
- `import numpy as np` (This allows us to manipulate lists and arrays.)
- `import os` (While we don't necessarily need it here since we're working on a Windows machine, it actually makes our final program usable both on Windows and MAC. Added flexibility is always a bonus!)
- `from lmfit.models import LinearModel, LorentzianModel, GaussianModel` (lmfit may not be installed on your machine. To install it, open the command prompt window (type `cmd` into the 'search' bar at the bottom of the windows screen), then type in `pip install lmfit` and hit enter. This library allows us to do more complicated fits without having to define the full fit function ourselves.)
- `from lmfit import fit_report` (This'll allow us to show the values of the fit parameters.)
- `from mpldatacursor import datacursor` (This is another nifty little library that allows you to extract  $x$  and  $y$  coordinates simply by clicking on the plot. If it is not installed, follow the instructions for `lmfit` but type `pip install mpldatacursor` instead of `lmfit`.)

Next, we'll have to tell Python where to look for the original data file. To do this, use

- `f = open(os.path.join('C:\\', 'filename.csv'), 'r')` (where you need to replace 'C:\\' with the actual location and 'filename' with the actual file name of your data file).

Note that you should direct Python to the csv (or comma-separated values) file. The 'r' tells Python that this is a read-only file (i.e. it won't try to write to it).

Next, we'll have to tell Python what the format of this data file is, and which columns we need (one for the time data and one for the absorption signal). This is how you do it:

- `flist = np.loadtxt(f, delimiter = ',', usecols=(3,4))` (Here, we're telling Python that it should look in the file we just imported (and called 'f'), that it should treat commas as delimiters, and that we only need the third and fourth column.)

It may be helpful to extract only some data from these columns, for example if your absorption dips are close to the center but you also recorded quite a bit of extra readings to the left and right of the dips. You want to narrow the data file down as much as possible to make the code run faster, but you don't want to cut off any valuable data. I'd suggest cutting it such that you have both dips in your data set but cut out any mode-hops that may be present in your data. If the full spectrum is mode-hop free, then please cut the data set such that it includes both dips but stops when they level out. For example, the following code selects the first 900 data points in both columns of 'flist':

- `timeData = flist[0:900, 0]`
- `absData = flist[0:900, 1]`

Now we're ready to plot the data. By including `datacursor()`, we'll be able to click on our data set:

- `plt.plot(timeData, absData)`
- `datacursor()`
- `plt.show()`

Select the 'Python' console (NOT 'IPython!') and run the code. You may have to fiddle with the limits you used in defining `absData` and `timeData`. Once the plot looks good to you, click on the minima of both dips and record the values in the space below. Calculate the time difference between the two minima and record it as well.

Next, we'll do the actual fit of our data to optimize these estimates. [Again, note that this may not be possible if the mode-hop free region of your ECDL is not wide enough.] To do this, use the following code to define the first Gaussian and modify it slightly to define a second Gaussian. Add both definitions to your Python script.

- `gauss1 = GaussianModel(prefix = 'G1_')` (This tells Python to use the Gaussian model. The prefix will identify all fit parameters as belonging to this first Gaussian. You'll have to change this for the second Gaussian!)
- `pars = gauss1(absData, x = timeData)` (You'll only need this line once, so don't repeat it for the second Gaussian.)
- `pars.update(gauss1.make_params())` (This will update everything. You'll need a similar command for your second Gaussian.)
- `pars['G1_center'].set(0.5, min = 0.1, max = 0.9)` (This tells Python that the center of the first Gaussian is most likely at 0.5, but could be anywhere between 0.1 and 0.9. You have to modify these values based on the values you extracted by clicking on the plot, and based on the width of the Gaussian.)
- `pars['G1_sigma'].set(0.005, min = 0.001, max = 0.01)` (In a similar structure, here you're telling Python what the width of the Gaussian is.)
- `pars['G1_amplitude'].set(-2, min = -3, max = 0)` (And here you're telling it what the amplitude is.)

To fully understand the significance of the parameters we just defined, look up the definition of a Gaussian and write it in the box below. Explain the meaning of the variables we introduced above. Also sketch a Gaussian function and indicate where you can find these parameters we just defined.

Next we'll need to tell Python which model we'd like to use. Since our absorption profile is a sum of two Gaussians, let's just add `gauss1` and `gauss2`. In addition, we'll also add an overall offset to account for the fact that the Gaussians don't level out at 0:

- `model = off + gauss1 + gauss2`

And lastly, we'll run the fit:

- `output = model.fit(absData, pars, x = timeData)`
- `print(output.fit_report(min_correl = 1e-13))` (This will print out the best values (including uncertainties) of your fitting parameters.)
- `plt.plot(timeData, output.best_fit, 'r')`
- `plt.plot(timeData, absData)`

- `plt.show()` (Here, we'll show both the original data and the fit on the same graph for easy comparison.)

From your fit, extract the position of the two centers and record them below (including uncertainties). Calculate the time difference between them (again, including uncertainties! Hint: Error propagation.) and, using the known frequency distance you calculated above, turn this into a conversion factor from time to frequency.

Next, load in the remaining four files that show high-resolution data of the individual lines into Python. Modify your code such that it expects only a single Gaussian and fit all four dips. Record the four widths in the box below (including uncertainties). Lastly, use the conversion factor to turn these extracted widths into actual frequency widths.

You may wonder why these lines are so broad. Naively, wouldn't we expect a very sharp absorption dip corresponding to the very narrow and sharp atomic resonance? Instead, we see this broad Gaussian shape! The reason for this broadening is the movement of the atoms in the glass cell that Doppler-shifts their resonance frequencies. Briefly review the Doppler effect and record key points in the box below.

\*

Remember that the perceived frequency is  $\omega = \omega_L \pm kv$ , where  $\omega_L$  is the actual laser frequency,  $\vec{k}$  the wavevector of the laser beam ( $k = 2\pi/\lambda$ ), and  $\vec{v}$  the velocity of the atoms. Just like the siren from a moving police car is Doppler shifted up or down, depending on whether the car is approaching or receding from you, the laser frequency is also shifted up or down. Explain whether the plus or minus sign corresponds to an atom moving toward the laser (note that this can also be written as  $\omega = \omega_L + \vec{k} \cdot \vec{v}$ ).

\*

Now you can understand why the width of the two Gaussians is so large: Since the atoms in the glass cell are at room temperature, they are moving around with quite some speed and in random directions. Thus, some atoms will move toward the laser beam while others will move away from it. This leads to a broadening of the spectral line because the laser frequency that the atom “needs” depends on its own speed. Because this broadening is caused by the Doppler effect, it is called *Doppler broadening*. Let’s estimate the temperature of the rubidium atoms in the glass cell from the measured width. First, what kind of temperature (in Kelvin) do you expect them to have?

The theoretical expression for the Gaussian width caused by the Doppler effect is given by  $\Delta\omega = \frac{2\omega_0}{c} \sqrt{2 \ln(2) \frac{k_B T}{m}}$ , where  $\omega_0$  is the transition frequency at rest,  $c$  is the speed of light,  $k_B$  is the Boltzmann constant,  $T$  the temperature (in kelvin!), and  $m$  the mass of the atom. Use your measured value of  $\Delta f$  to determine the temperature of the gas. Calculate it for four all the transitions you recorded (careful: isotopes!) and record all four values, including their uncertainties, in the box below. Is it allowed to average these four values? If so, record this average value (including its uncertainty) below. Do the temperatures agree with your expectations? If not, what could be a reasons for the disagreement? Could you test this somehow?

You may have noticed that the spectrum slowly drifts to one side. You may also have noticed that sometimes these drifts are more rapid, for example when someone opens the door or walks past the laser. As you push air through the cavity you change the density of the air inside the cavity. But changing the density also means changing the index of refraction of the air ever so slightly (in fact, the density and the refractive index are proportional to one another)! Explain why a changing index of refraction leads to a wobbling or shifting absorption signal. Hint: Think about standing waves in the extended cavity! If you want to know more about this phenomenon, have a look at the paper the Kleinert lab wrote in 2013 (Am. J. Phys. **81**, 929 (2013)).

To compensate for the fast drifts that are caused by changing air pressure you can wrap the whole ECDL in duct tape or place it in a sealed metal or plastic box. To further stabilize the laser diode, you can adjust the voltage to the PZT using a feedback loop and a proportional-integral (PI) controller. We will not do this here; if you want to know more talk to the Kleinert lab in which PI controllers are used to lock the laser frequency to the exact transition and compensate for all drifts.

We will now modify the setup to extract more detailed information about the individual hyperfine transitions. To get an idea of what we'll see, move the laser back to one of the large absorption dips and carefully rotate the glass cell. Observe the absorption profile. When the cell is almost perpendicular to the laser beam, you should see several sharper spikes appear in the broad Doppler dip. We will now investigate these spikes in more detail and learn where they come from.

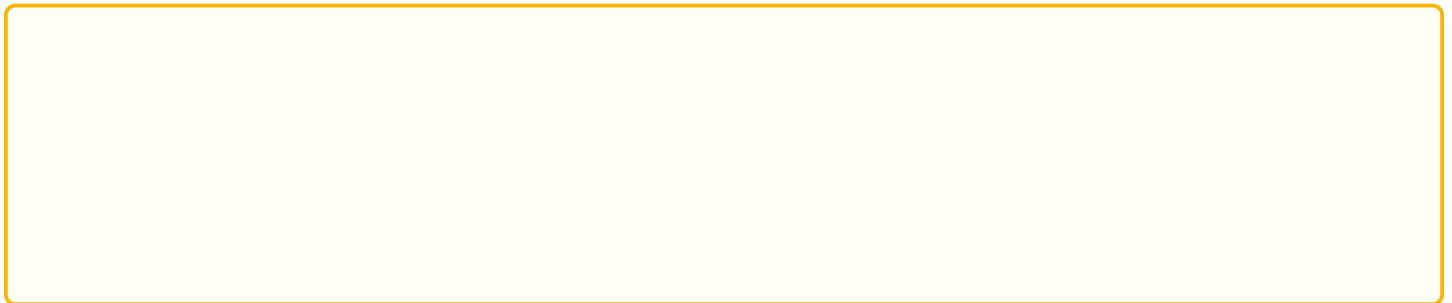
## 8 Doppler-free absorption spectroscopy

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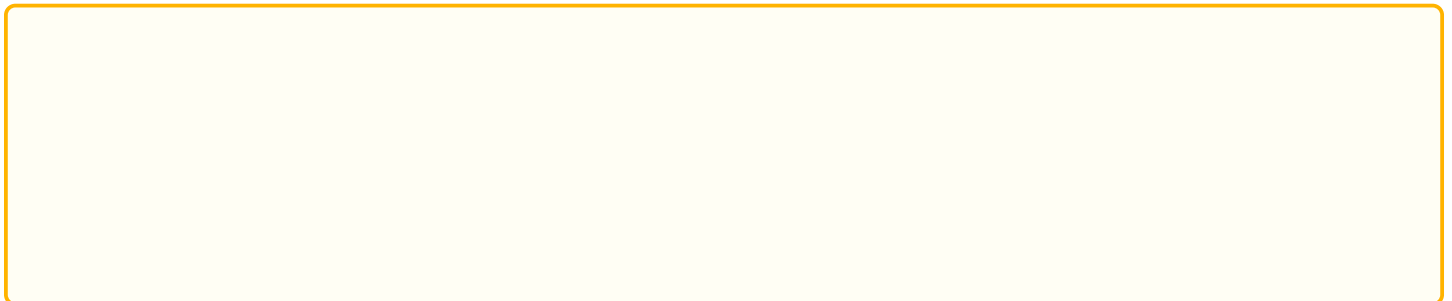
You just noticed that rotating the glass cell can make sharp spikes or peaks appear or disappear. Where do they come from? To understand this, let's think about what happens in the cell as you rotate it. Since the cell has no anti-reflection coating, we know that about 4-7 % of the light that hits the glass is reflected. The light that is reflected at the entrance surface will just be directed back toward the laser - that's why you initially made sure that the glass cell has some angle with respect to the laser, to avoid this unwanted feedback into the diode. The light that is reflected on the inside of the cell is much more interesting: It makes its way back into the cell and is now moving in the *opposite* direction of the initial beam. Thus, when there is only a very small angle between the cell and the incoming beam, the atoms will see *two* laser beams moving in *opposite* directions: The one coming from the laser, and the reflected one. Sketch this.



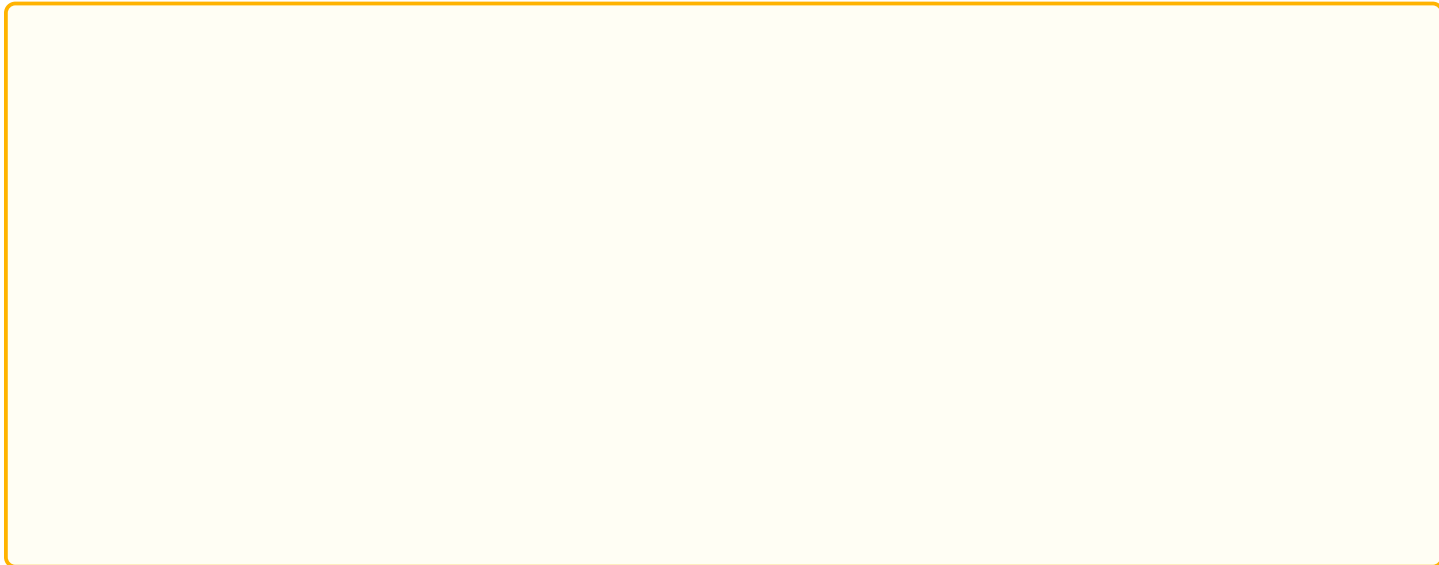
Imagine that you are an atom, and that you "see" these two laser beams. From which one would you absorb a photon? The answer depends on whether you are moving or not. Let's first assume that you, the atom, are moving toward the laser. Thus, the perceived frequency appears blue-shifted due to the Doppler effect, or slightly higher than the actual laser frequency because the Doppler shift compresses the wavefronts and makes the frequency appear higher. The laser beam from the reflection appears red-shifted (i.e. has a lower frequency) because the Doppler shift stretches the wavefronts and makes the frequency appear lower. Thus, as we scan the laser frequency from lower to higher frequencies, such an atom would first absorb photons from the laser beam coming directly from the laser, emit photons, and then absorb photons from the reflected laser beam as its frequency is shifted into resonance. Sketch this.



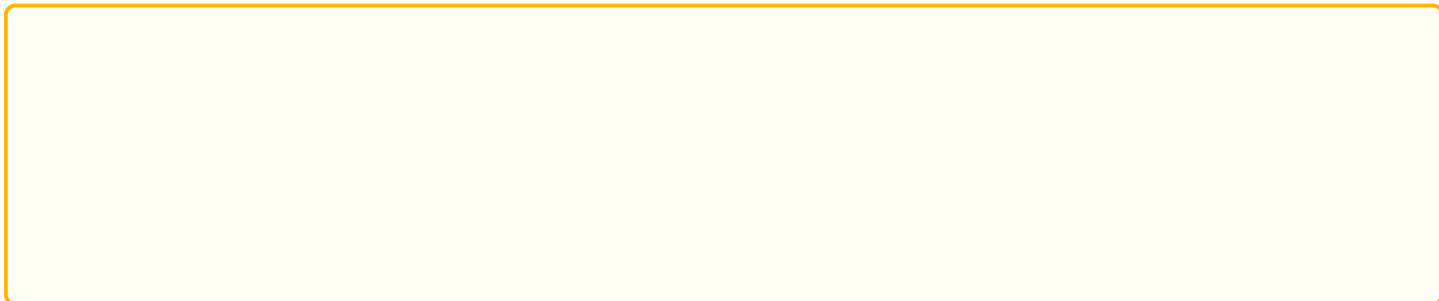
The situation is very similar for an atom moving away from the original laser beam. Explain what happens to such an atom as you scan the laser frequency.



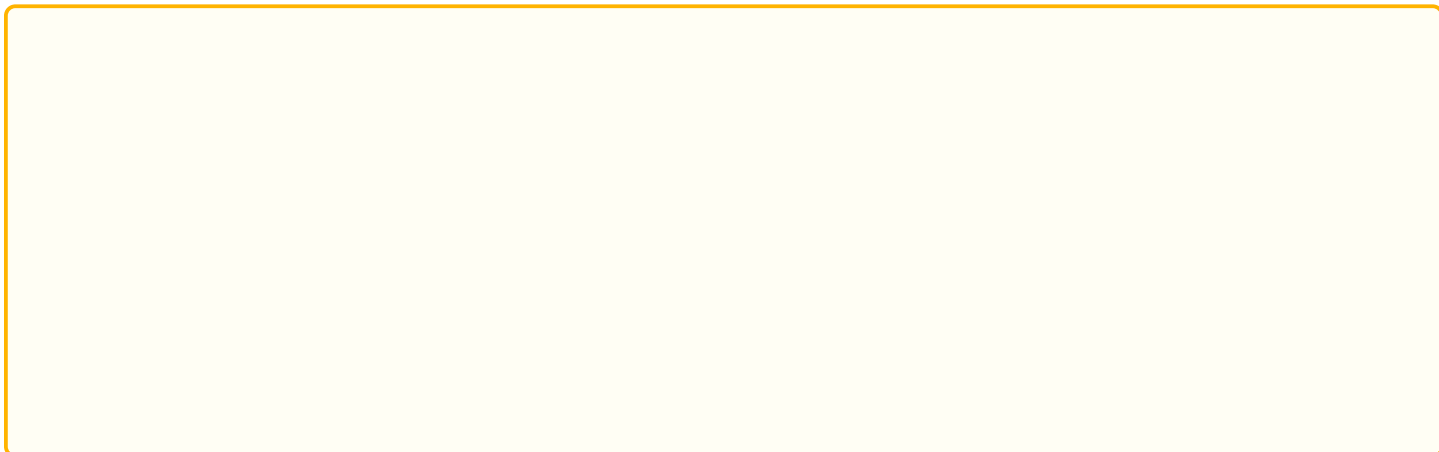
This is true for the majority of the atoms: Most of them will have some velocity component toward or away from the laser, and thus they will absorb photons from the original and the reflected beam at different laser frequencies during the frequency scan. Remember that this is also what leads to the broadened Gaussian profile! But there is *one* velocity class of atoms with a very specific velocity for which the situation is very different. Discuss with your group members what the velocity component along the direction of the laser beam for this special class of atoms must be and how this changes the above story. Record your hypothesis in the box below, and sketch at least two examples of how such an atom could be moving.



So, how does such an atom “decide” from which laser beam, the original one or the reflected one, it should absorb photons? Let’s look at an analogy: Imagine you are in the middle of a room and two friends are throwing soft balls at you, one from the left and one from the right. The one on the left throws a ball every minute. The one on the right throws a ball every second. Your task is to catch a single ball at some random time. Statistically, it is much more likely that you catch a ball that was thrown by your friend on the right, because they throw a lot more balls than your friend on the left. How does this analogy relate to the laser beams and the atoms?



How does that affect the absorption profile we record? To answer this question, think about what happens to an atom that “sees” two photons, one from the stronger *pump* laser and one from the weaker *probe* laser, and in particular what that means to the two photons. If you record photons from the weaker probe laser on a photodiode, do you expect the absorption to go up or down when the laser is in resonance with the atomic transition?

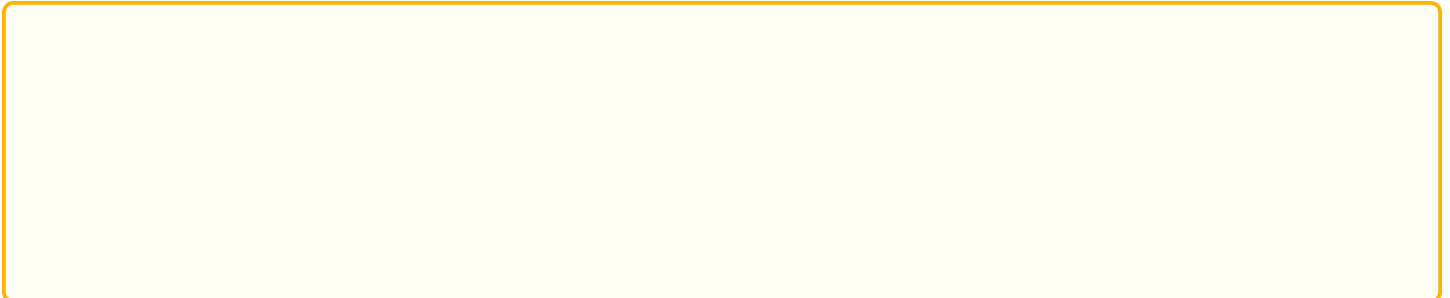




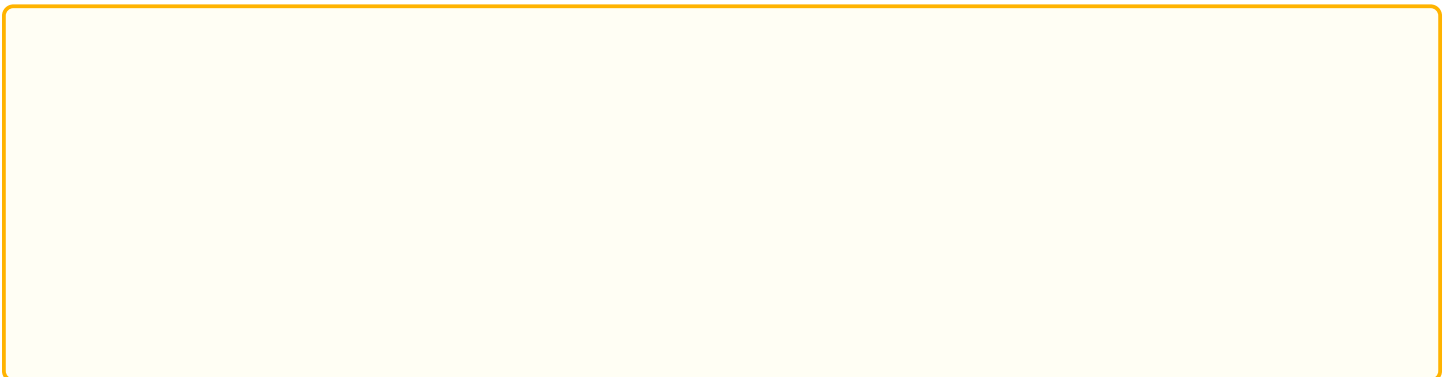
By simply tilting the glass cell, we generate two beams of approximately equal intensity. To improve the signal, place a half-wave plate and a polarizing beamsplitter cube between the laser and the glass cell. [If you need a refresher, watch the [VIDEOS](#) called [HOW TO MOUNT A HALF-WAVE PLATE](#), [HOW TO MOUNT A PBSC](#), and [HOW TO ALIGN A PBSC](#).] Adjust the half-wave plate such that the majority of the light passes through the PBSC. Block this beam after it has passed through the cell; this is our *pump* beam and it has fulfilled its purpose after it emerges from the cell. Using three mirrors (remember - you need two mirrors to walk a laser beam), redirect the weaker beam into the opposite end of the cell and adjust the mirrors such that the weak beam is almost counter propagating with the strong beam. You want to keep a small angle to avoid reflecting it directly back into the laser diode. Move the photodiode and record the weaker beam. Sketch the setup in the box below and explain why it makes sense to move the last two mirrors pretty far away from the glass cell to obtain a good signal.



Watch the spectrum on the oscilloscope and adjust the alignment until you see good spikes on top of the broad Gaussian dip (you should easily see three or more spikes). Note: You may have to move your photodiode a little as you adjust the weak *probe* beam. When you are satisfied with the alignment, record a trace for each of the two largest absorption dips. If you are curious, you may also adjust the laser frequency until you see the two smaller dips. In that case, record traces of these two as well. Record the file numbers in the box below.



To analyze your data, modify your code such that you can read in two files: One with the Doppler-free spikes and one without them (use the corresponding files from section 7). Create two lists and divide the `absData` from the Doppler-free file by the `absData` of the Doppler-broadened file. Note: You may have to scale your data such that the Doppler-broadened Gaussian profile has approximately the same depth in both of them. You also will have to shift your data horizontally by carefully cutting out the same regions. Plot this modified data set and sketch it in the box below. The two files have been correctly shifted and scaled when your data looks essentially flat, with just the spikes standing out. Attach a print out of your plot to this report.



Use the `datacursor` to find the center position of the spikes. Record these values in the box on the next page. Calculate all

possible differences between two spikes and convert them into actual frequency differences using your conversion factor from above.

Go back to the D Line data pdf and try to identify the origin of these spikes. What atomic transitions do they correspond to?

Chances are you could identify a few of these transitions but not all of them. If you analyze your data very carefully, you'll actually find that some of the spikes are located exactly halfway between two allowed transitions. These are the so-called *Cross-over resonance*. Briefly, they appear because a *moving* atom's perceived frequency can be up-shifted into resonance with one hyperfine transition for one of the laser beams and down-shifted into resonances with a different hyperfine transition for the counter-propagating laser beam. Discuss this with your group members and elaborate on it in the box below. Then identify the remaining spikes using this information and the data from the D Line data file.

And that's it! You made it successfully through the first module and are now qualified to perform the more advanced modules. Please leave us any comments, suggestions, or concerns in the box below, so that we can optimize this module for future student generations. Thanks!

