OPTIX Module 4 – Basic

Gaussian Laser Beams

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1 Objectives:

In this module you will learn about

- the mathematical description of Gaussian laser beams, in particular how the width of a Gaussian laser beam changes as it propagates;
- the two-dimensional Gaussian intensity profile and how it changes as the laser beam propagates;
- laser beam astigmatism and how to introduce it with a single lens.

Use this manual as you work through the module to keep track of your notes and thoughts. In addition, you may have to add a few printouts or refer to data tables or additional notes in your lab notebook. Make sure to add everything to the folder in which you keep this manual. Please note that this lab has no formal lab report. Instead, you will turn in and be graded on your notes in this manual and on the Jupyter notebook you'll submit after you have completed this lab.
This module is part of our Modern Physics curriculum and should take about 3 hours to complete.

2 Tests and assessment:

In preparation for this module, read through the whole manual and answer the questions that are marked with a *. You should also watch the `VIDEOs` that are posted on our website (`www.willamette.edu/cla/physics/info/NSF-OPTIX`). They are meant to accompany this manual and will show you some crucial steps of the module. When you come to lab, be prepared to discuss your answers to these questions with your classmates and your instructor.

In order to assess the success of this module, you will take a short assessment test before you start ("pre-assessment"), and another one after you have successfully completed this module ("post-assessment"). At this point you will also have the opportunity to provide us with feedback about the module that we will use to improve it for the next student generation. Thank you for your support!

3 Equipment:

You can find most of the equipment for this module in the box labeled 'HeNe Optics' that is located in one of the upper cabinets in the OPTIX lab. Standard equipment that is used for multiple modules is located in one of the other cabinets in the OPTIX lab. Please feel free to also ask your instructor for help. You will need:

- a helium-neon laser;
- a CCD camera (Thorlabs) with software;
- one 50 mm or one 100 mm focal length plano-convex lens;
- posts, post holders, bases, screws, and other standard hardware.

4 Introduction to Gaussian Laser Beams:

We often think of laser beams as infinitely thin rays, but closer inspection shows that they have a finite thickness. Real laser beams also don't stay collimated forever; they diverge slowly as they propagate. Remember that you briefly investigated this in `MODULE 1 - BASIC (Basic Optics - I)`. An even closer inspection reveals that the intensity of the beam is highest in the center and falls off radially as you move away from the center.

*** Based on this information, sketch an ideal laser beam (rays) and a real laser beam, both from the side and head-on. Point out differences and similarities between the ideal and the real beam.**

Imagine you measure the intensity of a Gaussian laser beam as you slowly move a detector through it from left to right. Such a graph is called an **intensity profile**: You're recording the intensity at a fixed height from left to right. We will do a similar measurement in a moment by using a CCD camera, which will allow us to record the full two-dimensional intensity profile at once. We'll see that the intensity profile is bell-shaped; it is a Gaussian (that's why laser beams are called Gaussian beams) and we can describe it with the two-dimensional function

$$I(x,y) = I_0 e^{-2x^2/w_x^2} e^{-2y^2/w_y^2}, \tag{4.1}$$

where x and y are the two directions perpendicular to the propagation of the laser beam (it is common to call the direction of propagation the z-axis).

*** Figure out what the parameters I_0, w_x, and w_y represent. Hint: Consider $x = y = 0$ to figure out what I_0 is, and $x = w_x/y = 0$ or $x = 0/y = w_y$ to figure out what w_x and w_y are. Note that these are the *definitions* for w_x and w_y!**

$I(0,0) =$

$I(w_x,0) =$

$I(0,w_y) =$

To gain some familiarity with this function, open a new Jupyter notebook and write a program that plots $I(x,0)$ in red and $I(0,y)$ in blue for $w_x = 2w_y = 1$ mm. Here's what your program should contain. Also don't forget to add comments and use markdowns!

- First, we have to import numpy and pyplot. Numpy will allow us to access the exponential function, while pyplot allows us to plot the function.
- Next, we'll define the function we want to plot. Note that x^2 in Python is written as `x**2`.
- Then we have to define a range for x and y. For this kind of Gaussian beam, you want to plot it over a range of about 10 mm, so ± 5 mm in both x and y. Think about the step size (1 mm? 0.1 mm? 0.01 mm?). The Python command for this is `np.arange(start, stop, step size)`.
- Lastly, plot the function for $x = 0$ or $y = 0$, respectively, and show both graphs in the same figure.

How do w_x and w_y affect the shape of the Gaussian? Can you generalize this, e.g. " as w gets smaller,...."

As you just saw in this example, w_x and w_y may be different, which means your laser beam is not round but rather elongated. In fact, this is actually quite common. While round laser beams are often desirable, for example when using lasers for micro-machining in industry, typical laser beams are rarely perfectly round. However, you can make them rounder by using either cylindrical lenses or prisms/prism pairs if your application requires this, as you will learn in `MODULE 1 - INTERMEDIATE`.

As the laser beam travels along the z-direction, its width in both the x and y-direction changes. Most often it simply increases as the beam diverges, but it is also possible that the width decreases for a while as the laser beam comes to a focus before diverging from there. Mathematically, that means that $w_x = w_x(z)$ and $w_y = w_y(z)$ are both functions of z, the direction of propagation of the laser beam. We can show that the width of a Gaussian laser beam changes with z as

$$w_x(z) = w_{0,x}\sqrt{1 + \left(\frac{z}{z_{R,x}}\right)^2},$$

where $w_{0,x}$ is the smallest possible width in the x-direction (also called the ***waist*** of the laser beam), and $z_{R,x}$ is the so-called ***Rayleigh Range*** given by $z_{R,x} = \frac{\pi w_{0,x}^2}{\lambda}$, where λ is the wavelength of the laser. A similar expression holds for the width in the y-direction, but keep in mind that the two waists $w_{0,x}$ and $w_{0,y}$ may be different.

Plot $w_x(z)$ in Python by modifying your code from page 2 and by assuming that $\lambda = 633$ nm, $w_{0,x} = 1$ mm, which are realistic values for the laser you are using. Sketch the function in the space below and identify $w_{0,x}$ and the Rayleigh Range in your sketch. Hint: Think about the value of $w(z_R)$ to figure out what the Rayleigh Range indicates.

Does a laser beam diverge more or less rapidly when you make its waist smaller? And how does changing the waist affect the Rayleigh Range? If you want your laser beam to remain collimated over a long distance, should its waist be large or small?

From your plot you may have noticed that the width of the Gaussian laser beam increases almost linearly for distances far away from the focus.

*** Show that this makes sense mathematically by derivation an approximation of $w_x(z)$ for large z.**

It is common to define the ***angle of divergence*** θ of a Gaussian beam as the angle between the horizontal and this linearly increasing waist at large distances.

Sketch the width of a Gaussian laser beam as a function of z in the space below and define θ in your sketch. Then derive an expression for this angle, assuming that the small angle approximation $\sin\theta \approx \tan\theta \approx \theta$ is valid.

Come up with a few general statements: Does a laser beam with a longer or shorter wavelength diverge more rapidly (i.e. have a larger angle of divergence)? Does a laser beam with a larger or smaller waist diverge more rapidly?

You can learn more about the mathematical description of Gaussian laser beams in our upper-level elective course Optics. For example, you will learn that in addition to the width w of the Gaussian beam you need to know its curvature R, and you'll be introduced to the complex q parameter and the so-called ABCD matrices that allow you to describe a Gaussian laser beam that encounters various optical elements like lenses or free-space propagation.

5 Measuring the Laser Beam Profile with a CCD Camera:

You are now ready to measure the beam profile of your HeNe laser using a CCD camera. Before we take this measurement, let's estimate the size of the laser beam.

Shine the laser onto a piece of paper, remove your goggles (while standing behind the laser) and roughly estimate its size in the x and y-direction, i.e. estimate w_x and w_y (including uncertainties).

$w_x \approx$

$w_y \approx$

Put your goggles back on.

Hold a detector card into the laser beam and again estimate the two widths. Record these estimates, including uncertainties.

$w_x \approx$

$w_y \approx$

You may have noticed that it is not easy to come up with good and consistent estimates. That's because the laser beam is quite intense and difficult to focus on, making it tricky for your eyes to estimate its size. In addition, the detector card saturates (making the beam look larger) and bleaches (the brightness changes when you hold the card into the beam for more than just a few seconds). Both of these effects make it difficult to get good and reliable estimates. That's where the CCD camera shines!

A CCD camera is a camera that uses semiconductor technology to record incident light/photons (CCD stands for *Charged Coupled Device*). The sensor of a CCD camera consists of several million small detectors called *pixels*. You can think of a pixel as a bucket that can hold photons. After the exposure, the number of photons in each pixel-bucket is then read out by the electronics in the camera and converted into an image of the light intensity that can be displayed on a computer screen. [Note that in reality, the photons are counted only indirectly. When a photon strikes the surface of a pixel, it creates an electron through the photoelectric effect. These electrons are then accelerated by electric fields and counted as an electric current.] Because the laser is quite intense and the CCD sensor is very sensitive, we risk overwhelming the CCD camera when we shine the laser directly onto its chip. In fact, there is a maximum number of photons that can be counted by a pixel on the CCD chip; once this maximum is exceeded, the pixel will just continue to read this maximum value; we say that the pixel is *saturated*. In our analogy, this would correspond to a full bucket. It doesn't matter if you add more photons to this bucket; the bucket can only hold so many of them.

Sketch a Gaussian intensity profile that is not saturated. Then sketch the same profile assuming that the pixels measuring the highest intensity are saturated. Explain the difference between the two sketches and come up with a general guideline that allows you to easily determine whether or not your CCD camera is saturated.

To avoid saturation, we have two options: We can either reduce the incident intensity of the laser beam, or we can reduce the amount of time that the sensor "sees" the incident light (this is called the *exposure time*).

How do you think the pixel count depends on the intensity? How does it depend on the exposure time?

To reduce the intensity of the laser beam without distorting its beam profile we will use a *neutral density filter*, which is similar to welder's glasses. It's also quite similar to your laser goggles, but instead of reducing the intensity within a narrow wavelength range, these filters reduce the intensity within the whole visible range.

In addition, we can also adjust the exposure time of the camera. To do this, make sure that the camera is connected to the computer with a USB cable. Find the `'T'`-icon on the desktop and double-click it. This should open the Thorlabs camera software by bringing up a small window with a `camera icon` in it; click on this `camera icon` and select the camera name. Click on the `film icon` on the far left side to start a live video feed. You should see a bright spot somewhere in the window. Confirm that this is indeed your laser beam by blocking/unblocking the beam with a piece of paper while observing the live feed.

Is the CCD camera saturated? Hint: The camera software offers you the option to look at a *line profile*. Find that option (it is one of the buttons) and use it to determine whether or not you are saturating the chip. Explain below and add a printout to this manual. What is the largest pixel value for this particular camera?

To adjust the exposure time of the camera, click on the `gear icon` to open the `settings`. Choose the shortest exposure time possible. Again observe the live feed. Are you saturating the CCD camera? Play with the exposure time and possibly add another neutral density filter until you see a nice Gaussian laser beam that fills out around 80% of the available range without saturating the CCD camera.

Now you are ready to measure the width of the laser beam in the x and y directions. Remember how these widths are defined (see page 2 of this manual), and use the line profile tool provided by the camera software to estimate the width.

Record the two values, including an estimate for their uncertainties, in the space below. The software allows you to measure this width in pixels. To convert it into actual length units, you can use that one pixel corresponds to 5.2 μm for this particular camera.

$w_x =$

$w_y =$

6 Measuring the width of a Gaussian laser beam as a function of z:

We will now confirm experimentally that the width of a Gaussian laser beam indeed changes with z as claimed on page 3 of this manual. To do this, mount a $f = 50$ mm or $f = 100$ mm lens in a lens mount, attach the mount to a post of good length, and slide it into a post holder that has been attached to a base. Place the lens directly behind the neutral density filter and arrange it such that the laser beam hits the lens perpendicularly and that it goes through the center of the lens. Review `MODULE 1 - BASIC (Optics Basics - I)` and watch the `VIDEO` called `HOW TO ALIGN A LENS` on our website. For this part of the exercise, it is *extremely* important that you align the lens well to avoid lens aberrations (we will look at lens aberrations in the last chapter of this manual!). Lock the lens down to the optics table and thoroughly tighten all screws on the lens mount. Using the detector card, find the focus behind the lens. Estimate over which range from the focus the laser beam diverges by about a factor of two or three; that's the range you want to cover on either side of the focus in the following exercise. The exact value of this range depends on the laser and the lens you are using, of course.

Record this range in the space below.

Discuss with your group members how you want to move the camera. For example, if the range you just determined is pretty large (several 10 cm or so), it is probably sufficient to move the camera by hand using that two neighboring holes on the optics table are separated by 1 inch, or using a meter stick to measure the distances from the focus. If, however, the range is pretty small (on the order of a cm or less), you may want to use a translation stage.

Write down your battle plan including how many data points you want to take and what the distance Δz between two data points is. Then check with your instructor (who can give you any additional equipment you may need).

Adjust the camera position until the laser beam appears as small as possible on the camera (i.e. until you are imaging the focus of the laser beam). Note: At this point you may have to add an additional neutral density filter that you can find in the box labeled 'Module 2' to further attenuate the laser beam!

Adjust the intensity and/or the exposure time so that the CCD isn't saturated and take the horizontal and vertical line profile of the laser beam. Measure the width in both directions, again remembering how this width is defined, and record it below (including an estimate of its uncertainty). This width is the smallest possible width corresponding to the waists.

$w_{0,x} =$

$w_{0,y} =$

Then move the camera a distance Δz that you agreed upon in your battle plan closer toward the laser.

Again measure the two widths. Record them, together with their uncertainties and the distance you just moved the camera.

Repeat until you have measured the widths at around six different locations. A data table works well for this task!

Then move the camera back to the focus and continue moving it by the same increment *away* from the laser.

Measure the two widths and repeat until you have at least five additional data points in your table.

In the end, your table should contain a total of at least 11 data points: one in the focus, and five on either side of it, spaced in equal increments.

Plot this data set in your Jupyter notebook. It might be helpful to know that you can create a list of data points by using square brackets, e.g. `positionData = [1,2,3,4,5]`, where you should replace the numbers with your actual measurements. In addition to your data, also plot the functional dependence you expect (i.e. $w_x(z)$ and $w_y(z)$) in the same graph, using the measured value of $w_{0,x}$ and $w_{0,y}$ in the focus. Does your data agree with your theoretical prediction? Discuss this with your group members and your instructor.

One possible reason for discrepancies could be that it is difficult to measure the waist in the focus precisely enough because of the finite size of the pixels. Another could be that your laser beam is not a perfect ideal Gaussian beam, so the functional dependence we are using may not match your laser's beam profile perfectly.

7 Lens aberrations:

To introduce lens aberrations into your Gaussian beam, position the camera such that it is *not* in the focus of the beam and tilt the lens slightly around a vertical axis without moving it back or forth. Observe what happens in the live feed of the camera and look at the line profiles.

Record your observations.

Then straighten the lens out again and drop it down a little, so that the laser beam now no longer goes through the lens' center but through the top part of the lens.

Do the line profiles change? If so, how? Does the focus shift (you can check that by moving the camera closer or farther away from the laser to see if that minimizes the width of the laser beam)? If so, how?

The distortions of the beam profile that you just observed are an example of *lens aberrations*. Shifting the lens introduces *spherical aberration*, while rotating it introduces *astigmatism*.

And that's it! You made it successfully through the first module and are now qualified to perform the more advanced modules. Please leave us any comments, suggestions, or concerns in the box below, so that we can optimize this module for future student generations. Thanks!